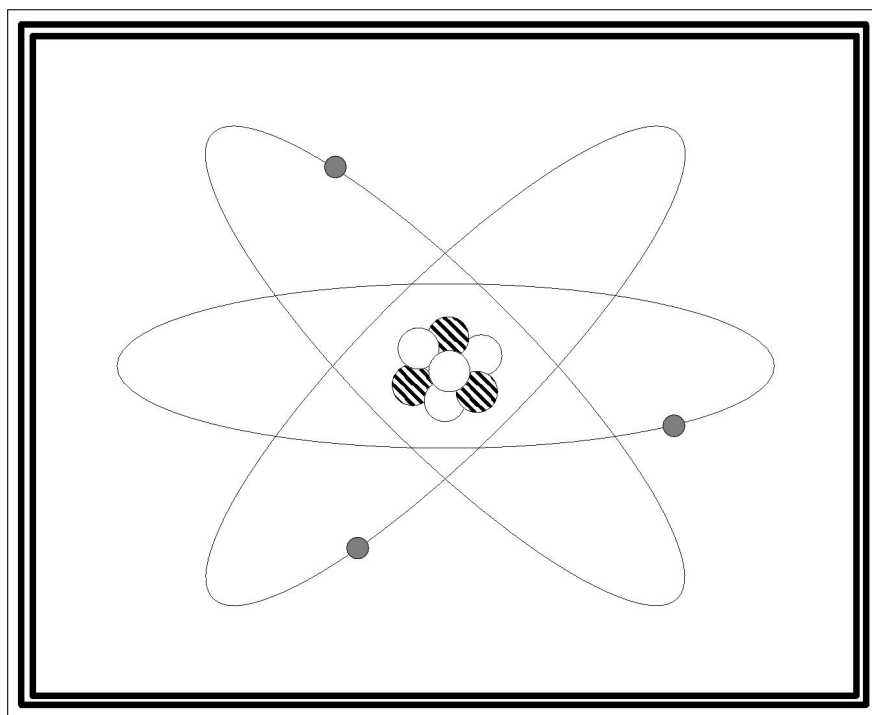


University of Northern British Columbia



Physics 100/110

Laboratory Manual

Winter 2018

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1. Introduction to Lab Reports

1.1. Lab Report Outline

Physics 1??

section L?

date

Lab #, Lab Title

Your Name

student #

lab partner's Name

The best laboratory report is the shortest intelligible report containing ALL the necessary information while maintaining university level spelling and grammar. Use full sentences and divide the lab into the sections described below.

Object - One or two sentences describing the aim of the experiment. Be specific in regards to what you hope to prove. (In ink)

Theory - The theory on which the experiment is based. This usually includes an equation which is to be verified. Be sure to clearly define the variables used and explain the significance of the equation. (In ink)

Apparatus - A brief description of the important parts and how they are related. In all but a few cases, a diagram is essential, artistic talent is not necessary but it should be neat (use a ruler) and clearly labeled. (All writing in ink, the diagram itself may be in pencil.)

Procedure - Describes the steps of the experiment so that it can be replicated. This section is not required.

Data - Show all your measured values and calculated results in tables. Show a detailed sample calculation for each different calculation required in the lab. Graphs, which must be plotted by hand on metric graph paper and should be on the page following the related data table. (If experimental errors or uncertainties are to be taken into account, the calculations should also be shown in this section.) Include a comparison of experimental and theoretical results. (Except graphs, this section must be in ink.)

Discussion and Conclusion - What have you proven in this lab? Support all statements with the relevant data. Was your objective achieved? Did your experiment agree with the theory? With the equation? To what accuracy? Explain why or why not. Identify and discuss possible sources of error. (In ink)

1.2. Lab Rules

Please read the following rules carefully as each student is expected to be aware of and to abide by them. They have been implemented to ensure fair treatment of all the students.

Missed Labs: A student may miss a lab without penalty if due to illness (a doctor's note is required) or emergency. As well, if an absence is expected due to a UNBC related time conflict (academic or varsity), arrangements to make up the lab can be made with the Senior Laboratory Instructor in advance. Students will not be allowed to attend sections, other than the one they are registered in, without the express permission of the Senior Laboratory Instructor. Failure to attend a lab for any other reason will result in a zero (0) being given for that lab.

Late Labs: Labs will be due 24 hours from the beginning of the lab period, after which labs will be considered late. Late labs will be accepted up until 1 week from the beginning of the lab. Students will be allowed 1 late lab without penalty after which all subsequent late labs will be given a mark of zero (0). Any labs not received within one week will no longer be accepted for grading and given a mark of zero (0).

Completing Labs: To complete the lab report in the lab time students are strongly advised to read and understand the lab (the text can be used as a reference) and write-up as much of it as possible before coming to the lab. We urge you to stay the entire lab time - it is provided for your benefit; if you run into problems writing up the lab, the instructor is there to help you.

Conduct: Students are expected to treat each other and the instructor with proper respect at all times and horseplay will not be tolerated. This is for your comfort and safety, as well as your fellow students. Part of your lab mark will be dependent on your lab conduct.

Plagiarism: Plagiarism is strictly forbidden! Copying sections from the lab manual or another person's report will result in severe penalties. Some students find it helpful to read a section, close the lab manual and then think about what they have read before beginning to write out what they understand in their own words.

Lab Presentation: Labs must be presented in a paper duotang. Marks will be given for presentation, therefore neatness and spelling and grammar are important. It makes your lab easier to understand and your report will be evaluated accordingly. It is very likely that, if the person marking your report has to search for information or results, or is unable to read what you have written, your mark will be less than it could be!

Contact Information: If your lab instructor is unavailable and you require extra help, or if you need to speak about the labs for any reason, contact the Senior Laboratory Instructor, Dr. George Jones, by email at gjones@unbc.ca, by phone at (250) 960-5169, or in his office, 10-2014.

Supplies needed:

- Clear plastic 30 cm ruler
- Metric graph paper (1 mm divisions)
- Paper duotang
- Lined paper
- Calculator
- Pen, pencil and eraser

Experiment 1: Newton's Second Law of Motion

UNBC Department of Physics

August 25, 2014

1. Introduction

Newton's second law of motion describes the acceleration of an object is affected by the net force acting on the object, and by the object's mass. This experiment will be performed with the aid of a computer interface system which will allow you to use the computer to collect, display, and analyze the data.

2. Theory

Newton's second law of motion states that the acceleration of an object or system is directly proportional to the net force acting on it and inversely proportional to its total mass. For motion in one dimension, Newton's second law takes the form

$$F = ma. \quad (1)$$

This is an example of a uniformly accelerated motion for which the following kinematic relations should hold true: The instantaneous velocity v of the system is given by

$$v = at + v_0, \quad (2)$$

where t denotes time and v_0 the initial velocity, and its displacement x is given by

$$x = \frac{1}{2}at^2 + v_0t + x_0, \quad (3)$$

where x_0 is the initial position.

Consider the system shown in Figure 2. We shall neglect for now the rotational motion of the pulley, the frictional force acting on the system, and the mass of the string connecting the two masses m and M . Applying Newton's second law to the motion of m gives

$$mg - T = ma. \quad (4)$$

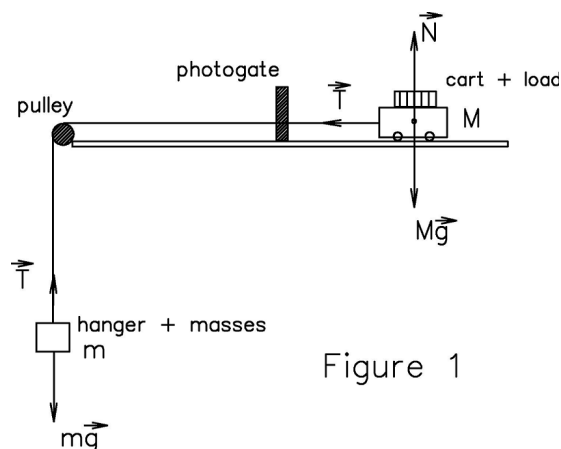


Figure 1

Applying Newton's second law to the motion of M (note that Mg and N cancel each other) gives

$$T = Ma. \quad (5)$$

Substituting (5) into (4) leads to

$$mg - Ma = ma, \quad (6)$$

which can be rearranged to give

$$a = \frac{m}{m + M}g. \quad (7)$$

This gives the value of the acceleration of either m or M . One way to interpret this equation is to say that the acceleration of the $(m + M)$ system is equal to the ratio between the net external force acting on it (mg) and its total mass $(m + M)$.

3. Apparatus

- computer running PASCO Capstone
- PASCO 850 Universal Interface
- dynamics cart and track system
- mass hanger and mass set (4x100g, 1x50g)
- pulley
- picket fence
- photogate

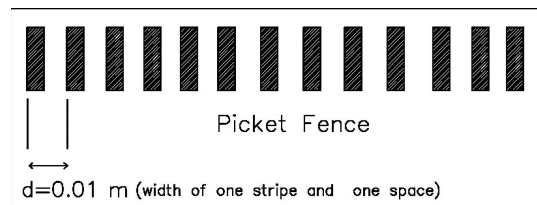


Figure 2

4. Procedure

1. Connect the photogate to the PASCO 850 Universal interface. Connect the interface to the computer using the USB cable. Now, launch the Capstone program.
2. The experiment uses a photogate to measure the motion of an object with uniformly spaced flags, which are normally provided by the black stripes of a picket fence mounted on the object whose motion is being studied. Set up the apparatus as shown in Figure 2. Make sure that the picket fence and track are free of dust and dirt, and that the mark on the picket fence are aligned with the photogate. If needed, ask for assistance from your laboratory instructor.
3. Follow the directions below to set up PASCO Capstone for data acquisition.
 - Switch on the 850 Universal Interface.
 - In Capstone, click on Hardware Setup to the left of the main window.
 - On the picture of the Interface, click on the digital port to which the Photogate is physically connected.
 - In the dropdown menu, click Photogate. Click Hardware Setup to close the window.
 - Click Timer Setup.
 - Click Preconfigured Timer. Click Next.
 - Make sure the Photogate is checked. Click Next.
 - Click Select A Timer. In the menu, click Picket Fence, then click Next.
 - Ensure that only Speed and Position are checked, then click Next.
 - Set Flag Spacing to 0.010m. Click Next.
 - Type a name for the timer and click Finish. Click Timer Setup to close the window.
 - Click Table & Graph.

4. Run 1: Place 150 g (one 100 and one 50) on the cart and make sure there are no slotted masses on the hanger. (The mass of the hanger, m , is 50g.) The mass of the cart (including the picket fence) is 510 g. Therefore $M = 0.510 \text{ kg} + \text{added masses}$. Record m and M in the data table.
5. Start data collection by clicking Record and immediately release the cart starting at one end of the track. Make sure to halt the cart after it passes through the photogate. Stop data collection by clicking Stop.
6. Click Select Measurement on one of the columns in the Data Table. Select Speed. Do the same on the y-axis of the Graph.
7. Look at the graph of Velocity vs Time on the screen. If the line is uneven, delete the data from the run and repeat data collection until the plot is a relatively smooth line.
8. Once a smooth line is recorded, click on Run # in the Legend to select the Graph,






then click the Fit Icon () and select Linear for the fit. Compare the equation of the line ($Y=mX+b$) with equation (2) (velocity is on the y-axis, time is on the x-axis, etc.) and then extract and record the value of the acceleration a_{ex} and the initial velocity $v_{0_{ex}}$.

9. Calculate and record the total mass, $M + m$, in the data table.
10. Using equation (1), calculate and record the force acting on the system, $F = mg$.
11. Then, using Equation (7), calculate and record the value of acceleration, $a_{th} = \frac{m}{m+M}g$.
12. Calculate and record the % difference between the theoretical and experimental values for acceleration,

$$\frac{|a_{th} - a_{ex}|}{|a_{th}|} \cdot 100\%$$

13. RUN 2: Move 50 g from the cart load to the mass hanger. ($M = 0.610 \text{ kg}$ and $m = 0.100 \text{ kg}$). Repeat steps 5 to 12. Note that the Run is automatically selected on the Table & Graph with the statistics.
14. RUN 3: Move an additional 50 g from the cart load to the mass hanger. ($M = 0.560 \text{ kg}$ and $m = 0.150 \text{ kg}$). Repeat steps 5 to 12. Note that the Run is automatically selected on the Table & Graph with the statistics.

15. RUN 4: Add 150 g to the cart load. ($M = 0.710$ kg and $m = 0.150$ kg). Repeat steps 5 to 12. Note that the Run is automatically selected on the Table & Graph with the statistics.
16. RUN 5: Add another 150 g to the cart load. ($M = 0.860$ kg and $m = 0.150$ kg). Repeat steps 5 to 12. Note that the Run is automatically selected on the Table & Graph with the statistics.
17. Click on the symbol  and uncheck Runs 2-5. The Graph will only show Run #1. On the Data Table, click Run # and select Run # 1. Click the Journal Snapshot Icon (). Click the Show Journal Icon (). You can now Print. Do this entire process for each Run.

5. Discussion and Conclusion

Comment on whether your results agree with Newton's Second Law. Explain. Outline and discuss at least two possible sources of error which may have occurred during the collection of the data and how they could have been avoided.

Data Table: Newton's Second Law

Run #	1	2	3	4	5
M (kg)					
m (kg)					
a_{ex} (m/s ²)					
$v_{0_{ex}}$ (m/s)					
$(m + M)$ (kg)					
F_{net} (N)					
a_{th} (m/s ²)					
% difference					

Date:

Instructor's Name:

Instructor's Signature:

Experiment 2: Atwood's Machine

UNBC Department of Physics

August 25, 2014

1. Theory

Atwood's machine consists of two masses connected by a cord and suspended by a pulley as shown in the Force Diagram. Suppose, for now, that the pulley and cord have negligible mass and that there is no friction acting on the system, and suppose that the masses start from rest. In response to the downward force of gravity, the heavier mass M_1 will fall, with an acceleration of magnitude a . If the cord is of fixed length, the lighter mass M_2 must rise with an acceleration of magnitude a as well. Applying Newton's second law to each mass gives

$$M_1g - T = M_1a \quad (1)$$

$$M_2g - T = -M_2a, \quad (2)$$

where we have chosen the positive direction to be downwards in each case. Notice also that the magnitude of the tension T is uniform throughout the cord. We can solve for a by subtracting equation (2) from equation (1),

$$(M_1 - M_2)g = (M_1 + M_2)a. \quad (3)$$

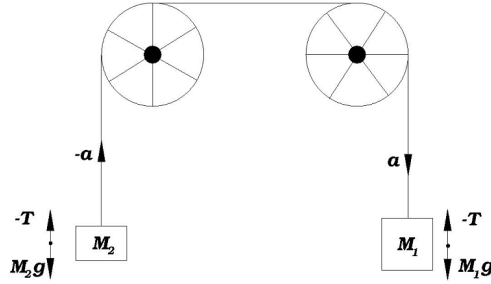
If we include the effects of the frictional force, f , then Equation (3) should be modified to:

$$(M_1 - M_2)g - f = (M_1 + M_2)a. \quad (4)$$

We also know that the acceleration, a , the distance, h , through which the heavier mass M_1 falls, and the time, t , taken for the mass M_1 to fall to the floor, are related by the equation for uniformly accelerated motion,

$$a = \frac{2h}{t^2}. \quad (5)$$

Force Diagram



2. Apparatus

- pulley System (Stand, Clamp, Mass Hangers, String, and Pulley)
- tape measure and stopwatch
- mass set (3x100g, 1x50g, 1x5g, 2x2g, 1x1g)

3. Procedure

1. Start with empty mass hangers. Since the masses of the hangers are equal (each has a mass of 50 g) the system should be in equilibrium. With one hanger touching the floor, measure the distance from the bottom of the other hanger to the floor; this is the distance h in Equation (5). The error δh in h will be 0.5 mm. Record these values in your lab report.
2. Let M_2 always be the mass which is initially on the floor. While holding the system still, add 0.160 kg to M_2 and 0.200 kg to M_1 . Including the masses of the hangers, the **total masses** are 0.210 kg and 0.250 kg, respectively. Record these values, along with the total mass, $M_1 + M_2$, in the data table.
3. Gently release the system from rest. With a stopwatch, measure the time it takes M_1 to travel the distance h from rest. Make three independent measurements of this time and record them as t_1 , t_2 and t_3 in the data table.
4. Knowing that you made three trials, $N = 3$, calculate and record the average time of fall, $\bar{t} = \frac{t_1+t_2+t_3}{N}$, the standard deviation of your time measurements, $\sigma_{s.d.} = \sqrt{\frac{(t_1-\bar{t})^2+(t_2-\bar{t})^2+(t_3-\bar{t})^2}{N-1}}$, and the error in \bar{t} , $\delta\bar{t} = \frac{\sigma_{s.d.}}{\sqrt{N}}$.
5. Using equation (5), determine and record the acceleration, $a = \frac{2h}{\bar{t}^2}$, of the system.

6. Calculate the error in a , $\delta a = a\sqrt{(\frac{1\cdot\delta h}{h})^2 + (\frac{-2\cdot\delta t}{t})^2}$ and record it.
7. Calculate the force acting on the system, $F = (M_1 - M_2)g$, and assume the corresponding error, δF , is zero since the error in the masses is negligible.
8. Repeat steps 3 through 7 five more times, each time moving 2 g from M_2 to M_1 (thus keeping the total mass of the system, $M_1 + M_2$, constant).
9. Use the data collected to plot a graph of Force versus Acceleration. Use the calculated values of δa to draw horizontal error bars to scale. Determine the best line through your points (It must intersect at least two points and have an equal number of points below and above the line) and use the error bars to find the maximum (steepest) and minimum (shallowest) slopes.
10. Determine the value of the best slope, m_{best} , using two points you DID NOT plot, (x_1, y_1) and (x_2, y_2) and the formula: $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$. Similarly find the values of m_{max} and m_{min} .
11. Find the error in the slope, δm , by calculating $m_{max} - m_{best}$ and $m_{best} - m_{min}$ and choosing the larger value.
12. Compare the equation of the line ($Y=mX+b$) with equation (4) (net force is on the y-axis, acceleration is on the x-axis, etc.).
13. Based on the comparison in the previous step extract the value of the effective frictional force, f , acting on your system as well as it's error, δf .

4. Discussion and Conclusion

Comment on whether your results agree with Newton's Second Law. Explain. Outline and discuss at least two possible sources of error which may have occurred during the collection of the data and how they could have been avoided.

Data Table: Atwood's Machine

M_1 (kg)						
M_2 (kg)						
$(M_1 + M_2)$ (kg)						
t_1 (s)						
t_2 (s)						
t_3 (s)						
\bar{t} (s)						
$\sigma_{s.d.}$ (s)						
$\delta \bar{t}$ (s)						
a (ms ²)						
δa (ms ²)						
F (N)						

Date:

Instructor's Name:

Instructor's Signature:

Experiment 3: Measurement of Acceleration Due to Gravity

UNBC Department of Physics

August 25, 2014

1. Introduction

Free fall is the motion of any object subjected to the gravitational field of the Earth. The magnitude of this acceleration can be developed as follows. Applying Newton's Law of Gravitation,

$$F = \frac{GM_{\text{E}}M}{(R + r)^2},$$

where M_{E} is the mass of the Earth, M is the test mass, G is the gravitational constant, R is the radius of the Earth and r is the radius of the test mass. Define g as

$$g = \frac{GM_{\text{E}}}{(R + r)^2}$$

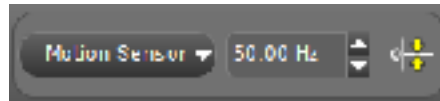
Thus, it can be seen that $F = Mg$, so g is an acceleration and is, in fact, the acceleration due to gravity. It can also be seen that g is independent of the mass M of the test object.

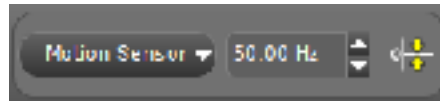
2. Equipment

- PASCO 850 Universal Interface
- PASCO Motion Sensor PS-2103A
- 16 mm steel ball
- tennis ball


3. Procedure

1. Open PASCO Capstone. Ensure that the 850 Universal Interface is on, and that the motion sensor is connected to a port on the 850 Interface. Move the switch on the front of the motion sensor to the stick figure. This is the long range setting.
2. In Capstone, click on Table & Graph. This is the template you will be using to collect data and to plot graphs.




3. In the sample rate selector , ensure that the sample rate is set to 50.00 Hz. Using the 16 mm steel ball, hold it 15 cm from the motion sensor as steadily as you can. Click Record and release the ball.
4. After the ball hits the ground, click Stop.
5. In the Capstone data table, click on <Select Measurement> and click on Velocity. The data will be loaded into the table, and the graph will appear.




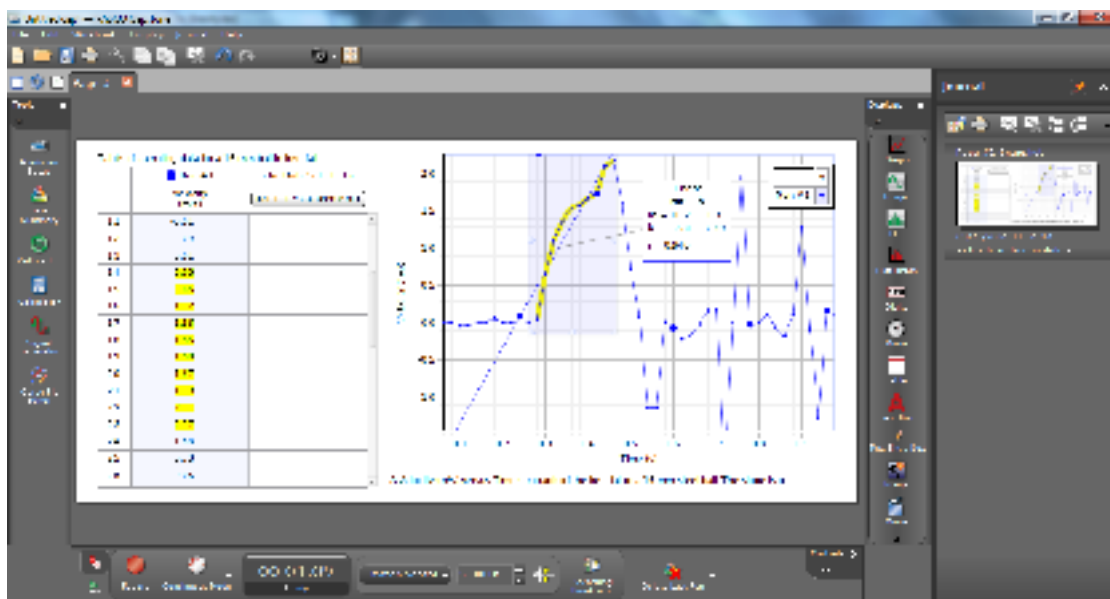
6. Click the highlighter , and a highlighter will appear on the graph. Resize and move this highlighter to select the roughly linear part of the graph. Do this for the first such region that appears on the graph. The other regions are due to the rebounds of the ball upon hitting the ground and are to be discarded. Highlighting this region will also highlight the corresponding data in the data table.



7. Now, click the curve fitting icon  and click on Linear: $mt + b$ to obtain a linear fit of the highlighted region. The slope should be close to the accepted value of $g = 9.81 \text{ m/s}^2$. If it is not, then there was some error in performing the release of the ball. In this case, you will need to perform more runs to generate a slope close to g .
8. Give the table a title by clicking in the region where it says to enter a title. Do the same for the graph.



9. Click the Journal icon  to take a snapshot of this page in Capstone. Click the Show Journal icon next to the camera icon to show the journal in the right of the page. Now, you can print the sheet using the printer icon (see Figure ??).
10. Obtain 5 more such runs on separate pages performing all the tasks in the previous steps.



11. At this point you will need to find the standard deviation involved in the values you obtained for g . Ignore the errors in the slopes of the graphs which Capstone calculated.
12. Find the mean as $\bar{g} = \frac{g_1+g_2+g_3+g_4+g_5+g_6}{6}$.
13. Now find the standard deviation as:

$$\sigma_{s.d.} = \sqrt{\frac{(g_1 - \bar{g})^2 + (g_2 - \bar{g})^2 + (g_3 - \bar{g})^2 + (g_4 - \bar{g})^2 + (g_5 - \bar{g})^2 + (g_6 - \bar{g})^2}{6 - 1}}. \quad (1)$$

14. Finally, find the error in the mean as $\delta\bar{g} = \frac{\sigma_{s.d.}}{\sqrt{6}}$.
15. Repeat for tennis ball.

4. Discussion and Conclusion

Comment on whether your results support that the acceleration due to gravity of an object is independent of the object's mass and composition. Explain. Outline and discuss at least two possible sources of error that may have occurred during the collection of data and how they could have been minimized or avoided.

Experiment 4: Hooke's Law and Simple Harmonic Motion

UNBC Department of Physics

August 25, 2014

1. Introduction

Hooke's law describes the motion of a spring. This law is valid only in the case of an ideal (i.e., perfectly elastic) spring. However, we can do experiments with real springs which exhibit ideal behavior over a limited range of extension. We will also study one of the most interesting types of motion in nature, simple harmonic motion. Such motion takes place whenever the force acting on a system takes the form of a restoring force of the form given below. Examples include the oscillatory motion of a mass-spring system and the periodic motion of a pendulum.

Consider the mass-spring system in the following force diagram. The spring elastic force acting on the mass is a restoring force having the form

$$F = -kx, \tag{1}$$

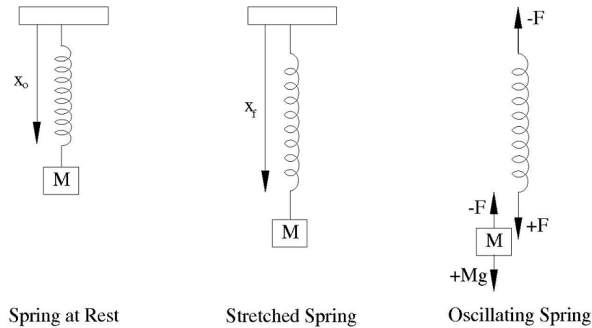
where $x = (x_f - x_0)$ is the amount of stretching undergone by the spring away from its unstretched position x_0 , and k is a constant characteristic of the spring called the force constant. The minus sign indicates that the tendency of the spring force is to oppose this stretching and push the system back toward the unstretched position (hence the term restoring force). Equation (1) is called Hooke's law, which we will apply to two cases.

Static case

When the system is in equilibrium, equation (1) is simply a statement that the amount of extension x is proportional to the force applied to the spring which in this case is equal in magnitude to the weight of the suspended mass,

$$|F| = Mg = k(x_f - x_0) = kx \tag{2}$$

Note: We have neglected the small mass of the spring compared to the suspended mass.



Dynamic case

If the mass is pulled down from its equilibrium position by a small distance and gently released, then what follows is oscillatory, periodic motion (or simple harmonic motion) wherein the block moves up and down around the equilibrium position. Such motion is characterized by its period of oscillation T , which is given by

$$T = 2\pi\sqrt{\frac{M}{k}} \quad (3)$$

2. Apparatus

- Hooke's Law apparatus (stand, ruler, mass hanger)
- stopwatch
- 2 Springs (small and large)
- mass Set (1x100g, 5x20g, 1x10g)

3. Procedure

Static Case:

1. Using the larger spring and with no mass on the hanger, measure the position of the pointer attached to the spring, x_0 . The error δx_0 in x_0 will be 0.5 mm. Record these measurements in Table 1.
2. Add 10 g to the mass holder whose mass is 10 g. Record the total mass acting on the spring, $M = 0.020$ kg, in Table 1. Calculate and record the force being exerted on the spring due to gravity, $F = Mg$. Assume the corresponding error, δF , is zero, since the error in the masses is negligible.

3. Measure the pointer's new position, x_f . Since the measuring device being used is the same as that used to measure x_0 , the error δx_f in x_f , will be the same as δx_0 , 0.5 mm.
4. Calculate and record the total displacement, $x = (x_f - x_0)$. Find the error in displacement, $\delta x = \sqrt{(\delta x_0)^2 + (\delta x_f)^2}$.
5. Repeat steps 2 through 4 nine times, each time adding 10 g to the hanger, and record your results in Table 1.
6. Use the data collected to plot a graph of Force versus Displacement, and test for linearity. Determine the best line through your points (It must intersect at least two points and have an equal number of points below and above the line).
7. Determine the value of the best slope, m_{best} , using two points you DID NOT plot, (x_1, y_1) and (x_2, y_2) and the formula: $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$.
8. Compare the equation of the line ($Y = mX + b$) with equation (2) (force is on the y-axis, displacement is on the x-axis, etc.).
9. Based on the comparison in the previous step, deduce the value of the spring constant and call it k_{Large} .
10. Repeat steps 1 through 9 for the smaller spring, and call the spring constant you deduce in this case, k_{Small} . Record the experimental data in Table 2.

Dynamic Case

11. Now you are to measure the period of oscillation of the simple harmonic motion of the mass-spring system and compare it to equation (3). First measure and record the mass of the **larger** spring, M_{spring} in Table 3.
12. You will begin by adding 100 g to the 10 g hanger. Record the total mass, $M = 0.110$ kg in Table 3. Assume the corresponding error, δM , is zero since the error in the masses is negligible.
13. Gently set the system in motion by slightly lifting the mass and then releasing it. Try your best to obtain up-and-down oscillations with no twisting or swaying of the spring (this would complicate the motion and affect the measured values). Also, make sure that the oscillations occur inside the region where Hooke's law applies (otherwise equation (3) would cease to be valid). Wait a few moments to ensure the stability of the oscillation then measure the time t for 10 complete oscillations (cycles). Assume an error $\delta t = 0.09$ s in t . (0.09 s should be a reasonable error, reflecting how accurately and how fast you are timing the start and end of the 10 cycles).

14. Deduce the value of the period, $T = \frac{t}{10}$. Calculate the error $\delta T = \frac{\delta t}{10}$ in T . Continue to record your data and results in Table 3.
15. Calculate and record T^2 . You can find its corresponding error using $\delta(T^2) = T^2 \cdot \frac{2 \cdot \delta T}{T}$.
16. Repeat steps 11 through 15 five times, each time adding 20 g, and record your results in Table 3.
17. Equation (3) can be rearranged algebraically to give:

$$M = \frac{k}{4\pi^2} T^2 \quad (4)$$

Use the data collected to plot a graph of Mass vs. Period² and test for linearity. Use the calculated values of δT^2 to draw horizontal error bars to scale. Determine the best line through your points (It must intersect at least two points and have an equal number of points below and above the line) and use the error bars to find the maximum (steepest) and minimum (shallowest) slopes.

18. Determine the value of the best slope, m_{best} , using two points you DID NOT plot, (x_1, y_1) and (x_2, y_2) and the formula: $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$. Similarly find the values of m_{max} and m_{min} .
19. Find the error in the slope, δm , by calculating $m_{max} - m_{best}$ and $m_{best} - m_{min}$ and choosing the larger value .
20. Compare the equation of the line ($Y=mX+b$) with equation (4) (mass is on the y-axis, period squared is on the x-axis, etc.).
21. Based on the comparison in the previous step extract the value of the spring constant, $k = 4\pi^2 \cdot m$. Also find its error, $\delta k = 4\pi^2 \cdot \delta m$.
22. In addition, find and record the value of the best y-intercept, b , the maximum y-intercept, b_{max} , and the minimum y-intercept, b_{min} . Find δb by calculating $b_{max} - b_{best}$ and $b_{best} - b_{min}$ and choosing the larger value.

23. If the effective mass of the spring mass, M_{eff} , is included, it can be shown that equation (4) should be replaced by the more accurate expression

$$M + M_{\text{eff}} = \frac{k}{4\pi^2} T^2, \quad (5)$$

which can be rearranged to give

$$M = \frac{k}{4\pi^2} T^2 - M_{\text{eff}} \quad (6)$$

Compare Equation (6) to the straight line equation and determine the experimental value of the effective mass of the spring, $M_{\text{eff}_{ex}} = -b$, and its error, $M_{\text{eff}_{ex}} = \delta b$. Compare M_{eff} to the expected ranges of values, $M_{\text{spring}}/3 < M_{\text{eff}} < M_{\text{spring}}/2$.

4. Discussion and Conclusion

Comment on whether your results agree with Hooke's Law. Explain. Outline and discuss at least two possible sources of error which may have occurred during the collection of the data and how they could have been avoided.

Table 1: Hooke's Law for the Larger Spring

$x_o \pm \delta x_o =$			
M (kg)	F (N)	$x_f \pm \delta x_f$ (m)	$x \pm \delta x$ (m)

Date:

Instructor's Name:

Instructor's Signature:

Table 2: Hooke’s Law for the Smaller Spring

$x_o \pm \delta x_o =$			
M (kg)	F (N)	$x_f \pm \delta x_f$ (m)	$x \pm \delta x$ (m)
Date:			
Instructor’s Name:			
Instructor’s Signature:			

Table 3: Dynamic Case for the Larger Spring

$M_{spring} =$			
M (kg)	$t \pm \delta t$ (s)	$T \pm \delta T$ (s)	$T^2 \pm \delta T^2$ (s ²)
Date:			
Instructor's Name:			
Instructor's Signature:			

Experiment 5: The Simple Pendulum

UNBC Department of Physics

August 25, 2014

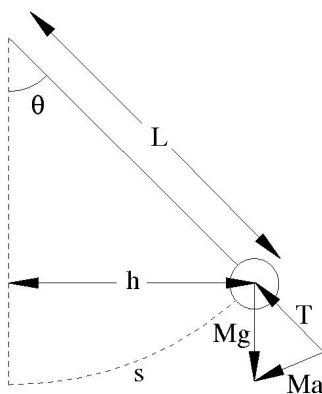
1. Introduction

A simple pendulum consists of a mass M attached by a string to a fixed point. The mass will execute periodic motion along a circular arc when released from rest to one side of the vertical. The position of the mass may be given in terms of the length L of the string and the angle θ of the string as measured from the vertical (see Figure 1). The component a_s of the acceleration of the mass M is in the direction perpendicular to the line of the string is given by Newton's second law as

$$F = Mg \sin \theta = Ma_s. \quad (1)$$

Notice that we have taken the component of the weight along the direction perpendicular to the line of the string. For large angles, further analysis of the motion of the mass using Equation (1) is complicated. However, if we restrict our investigation to small angles, we may make approximations which simplify matters. For small angles, we have

$$\sin \theta = h/L \approx s/L, \quad (2)$$



where θ is in radians. Substituting equation (2) into equation (1) gives an expression for the magnitude of a_s ,

$$a_s \approx (g/L)s. \quad (3)$$

Whenever acceleration is proportional and opposite to displacement, the motion is called simple harmonic motion. This type of motion occurs for the simple pendulum when the displacement, s , is small.

We can construct a relationship between the period T and the length L of the pendulum by using the method of dimensional analysis. In this method, we ensure that the dimensions (units) are the same on both sides of the equation. We expect the period to depend on the acceleration due to gravity, g , because we know that no oscillations would occur if g were zero. We therefore expect a relationship of the form

$$T = cL^\alpha g^\beta, \quad (4)$$

where c is a numerical constant having no dimensions, and the exponents α and β are to be found using dimensional analysis.

It can be shown that, in order that equation (4) provides a good estimate for the period T of the simple pendulum, the magnitude of s/L should typically be no bigger than about 0.1.

2. Apparatus

- pendulum system (stand, string and bob)
- meter stick
- stopwatch

3. Procedure

Advance Preparation

1. Using the method of dimensional analysis (see the Appendix), determine the values of the exponents α and β in equation (4). Record your equation (4) with these values of α and β .
2. Explain how you can test your equation using graphical analysis. What should be plotted on the “ y -axis”? What should be plotted on the “ x -axis”? Write your

expression in the standard form $y = mx + b$. What is your expression for the slope m ?

3. Spend a few moments familiarizing yourself with the apparatus and the method of adjusting the length of the string.
4. Start by adjusting the string to have a length of 20 cm from the fixed point to the *bottom of the mass*, and record this length, L , in the data table. Also, record the error in the length, $\delta L = 0.5$ mm.
5. Pull the mass to one side of the vertical. Make sure that the ratio s/L (see Figure 1) is no bigger than 0.1. Release the mass from rest, and start the stopwatch at the same instant. Allow the bob to oscillate through 10 cycles, and then stop the stopwatch at the instant that the mass reaches its greatest deflection from the vertical. Record your value for the total time, t_1 , of the 10 cycles.
6. Repeat step 5 four times with the same length of string. Record the times for 10 cycles as t_2 , t_3 , t_4 , and t_5 .
7. Knowing that you made five trials, $N = 5$, calculate and record the average time of fall, $\bar{t} = \frac{t_1+t_2+t_3+t_4+t_5}{5}$, the standard deviation of your time measurements, $\sigma_{s.d.} = \sqrt{\frac{(t_1-\bar{t})^2+(t_2-\bar{t})^2+(t_3-\bar{t})^2+(t_4-\bar{t})^2+(t_5-\bar{t})^2}{5-1}}$, and the error in \bar{t} , $\delta\bar{t} = \frac{\sigma_{s.d.}}{\sqrt{5}}$.
8. By dividing \bar{t} by the number of oscillations in each trial, it is possible to find the average period, $T = \frac{\bar{t}}{10}$. Calculate the error in T , $\delta T = \frac{\delta\bar{t}}{10}$.
9. Repeat steps 4 through 8 for lengths of 25, 30, 35, 40, 45, and 50 cm. Record all your values in the data table.
10. Calculate and tabulate your values for what you determined y , δy , x and δx to be in step 2.
11. Plot a graph of y versus x . Use the calculated values of δx and δy to draw horizontal and vertical error bars respectively. Determine the best line through your points (It must intersect at least two points and have an equal number of points below and above the line) and use the error bars to find the maximum (steepest) and minimum (shallowest) slopes.
12. Determine the value of the best slope, m_{best} , using two points you DID NOT plot, (x_1, y_1) and (x_2, y_2) and the formula: $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$. Similarly, find the values of m_{max} and m_{min} .
13. Find the error in the slope, δm , by calculating $m_{max} - m_{best}$ and $m_{best} - m_{min}$ and choosing the larger value.

14. Using the expression for slope you found in step 2 and your values for the slope and its error, determine the numerical value of c and its error. Find the percent difference between this value and the expected value of 2π .
15. Examine your graph. If you have been careful enough in collecting your data, you may well find that $\mathbf{x - intercept} \neq \mathbf{0}$. After thinking about it, you will agree that the distance to the center of the mass is the correct length to use in Equation (4). Therefore you should indeed expect that the x-intercept will be nonzero. So Equation (4) should be modified to read:

$$T = c(L - r)^\alpha g^\beta \quad (5)$$

where L is the length of the string and r is the radius of the weight. Measure and record the radius of the weight r , at the end of the string.

16. Find and record the value of the best x-intercept, $X_{intercept}$, the maximum x-intercept, X_{max} , and the minimum x-intercept, X_{min} . Find $\delta X_{intercept}$ by calculating $X_{max} - X_{best}$ and $X_{best} - X_{min}$ and choosing the larger value. Find the percent difference between the radius of the ball and the x-intercept.

4. Discussion and Conclusion

Comment on whether your results support the constructed relationship between period and length. Explain. Outline and discuss at least two possible sources of error which may have occurred during the collection of the data and how they could have been avoided.

Data Table: Simple Pendulum

L (m)							
δL (m)							
t_1 (s)							
t_2 (s)							
t_3 (s)							
t_4 (s)							
t_5 (s)							
\bar{t} (s)							
$\sigma_{s.d.}$ (s)							
$\delta \bar{t}$ (s)							
T (s)							
δT (s)							

Date:

Instructor's Name:

Instructor's Signature:

Experiment 6: Elastic and Inelastic Collisions

UNBC Department of Physics

August 25, 2014

1. Introduction

This experiment studies the collision of two objects, a lighter object that has mass m and speeds v and v' before and after the collision, and a heavier object that has mass M and speeds V and V' before and after the collision. Only linear motion is considered in this experiment; consequently, velocities are speeds multiplied by \pm signs.

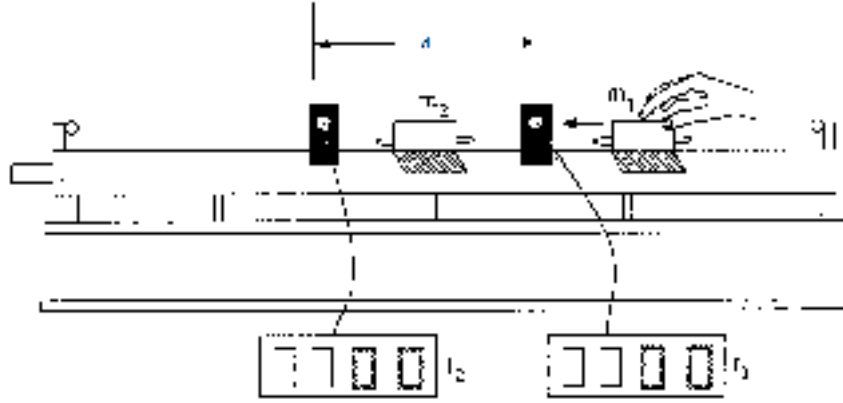
A collision usually happens over a very short duration of time, and the collision force is large, i.e., the collision force is an example of an impulsive force. The linear momentum \vec{p} of an object is the vector quantity that is the product $m\vec{v}$, where m is the mass of the object and \vec{v} is the velocity of the object. Because an internal impulsive force dominates during collisions, total linear momentum is conserved, i.e., the vector sum of the momenta of the objects immediately before a collision, $\vec{p}_{\text{initial}} = m\vec{v} + M\vec{V}$, equals the vector sum of the momenta of the objects immediately after the collision, $\vec{p}_{\text{final}} = m\vec{v}' + M\vec{V}'$. The relation

$$\begin{aligned}\vec{p}_{\text{initial}} &= \vec{p}_{\text{final}} \\ m\vec{v} + M\vec{V} &= m\vec{v}' + M\vec{V}'\end{aligned}$$

is called conservation of momentum.

The kinetic energy of an object that has mass m and speed v is $\frac{1}{2}mv^2$. If the sum of the kinetic energies of the objects immediately before a collision, $K_{\text{initial}} = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$, equals the sum of the kinetic energies of the objects immediately after the collision, $K_{\text{final}} = \frac{1}{2}mv'^2 + \frac{1}{2}MV'^2$, then the collision is said to be an elastic collision. If $K_{\text{initial}} \neq K_{\text{final}}$, then the collision is said to be an inelastic collision. If the two objects stick together after the collision, then the collision is said to be a completely inelastic collision.

Figure 1 is a diagram of the experiment.



Collision 1

Before the collision 1, m moves to the right with speed v and M is stationary, $V = 0$. After the collision, m moves to the left with speed v' and M moves to right with speed V' . Consequently,

$$\begin{aligned}\vec{p}_{\text{initial}} &= mv \\ \vec{p}_{\text{final}} &= -mv' + MV' \\ K_{\text{initial}} &= \frac{1}{2}mv^2 \\ K_{\text{final}} &= \frac{1}{2}mv'^2 + \frac{1}{2}MV'^2.\end{aligned}\tag{1}$$

Collision 2

Before the collision 2, m moves to the right with speed v and M is stationary, $V = 0$. After the collision, m and M stick together, and hence both move to the right with the same speed v' . Consequently,

$$\begin{aligned}\vec{p}_{\text{initial}} &= mv \\ \vec{p}_{\text{final}} &= (m + M)v' \\ K_{\text{initial}} &= \frac{1}{2}mv^2 \\ K_{\text{final}} &= \frac{1}{2}(m + M)v'^2.\end{aligned}\tag{2}$$

2. Equipment

1. PASCO track
2. 2 collision carts with picket fences
3. 2 photogates
4. 500 g mass
5. PASCO 850 Universal Interface

3. Procedure

1. Set up the apparatus as shown in Figure 1.
2. Launch Capstone, then switch on the 850 Universal Interface.
3. In Capstone, click on Table & Graph.
4. Click on Hardware Setup to the left under Tools.
5. Click on the first port on the 850 diagram which is physically connected to a photogate. In the drop-down menu, click on Photogate. Do the same with the other port. Click on Hardware Setup to collapse the window.
6. Click on Timer Setup under Tools. Leave the first step (1) untouched. Click Next.
7. Under 2, both photogates should be checked. If not, check them. Click Next.
8. In 3, click Select a Timer, then click Collision (Double Flag).
9. In 4, ensure that Block-Block Time, Gate 1, Block-Block Time, Gate 2, Speed in Gate 1 and Speed in Gate 2 are checked, with all others unchecked. Click Next.
10. In 5, enter the spacing of your photogates. You can set this on the track using the embedded metre rule. Click Next.
11. In 6, enter a name for the timer. Click Finish. Click Timer Setup to collapse the window.
12. Mass M is the 0.520 kg cart with plunger together with the 0.500 kg mass. Mass m is the other cart, which has a mass of 0.505 kg.

Collision 1

13. Now place M at rest between the photogates with the plunger to the right, and with the rightmost mark on the picket fence is about 1 cm to the left of the right gate. Place m such that the **Velcro is on the left**, and such that the cart is completely to the left of the left gate.
14. Click Record, and give m a push to the right towards M . Push it hard enough that it magnetically collides with M , but not so hard that m and M touch. Mass M should move completely past the right gate. Mass m will recoil in the opposite direction, and should move back completely past the left gate.
15. Click Stop.
16. Velocity graphs for each cart can be displayed by clicking <Select Measurement> on the y-axis and then selecting the desired velocity.
17. Notice that speeds decrease as the carts pass through the photogates; this is because of friction. Speeds v , v' and V' are the appropriate maximum speeds on the graphs, as these are the speeds immediately before and after the collision.
18. Record v , v' and V' .
19. Complete the first row of the Observations table.

Collision 2

20. Now place M at rest between the photogates with the plunger to the right, and with the rightmost mark on the picket fence is about 1 cm to the left of the right gate. Place m such that the **Velcro is on the right**, and such that the cart is completely to the left of the left gate.
21. Click Record, and give m a push to the right towards M . Push m hard enough that m and M collide, stick together, and move completely past the right gate.
22. Click Stop.
23. Display the velocity graphs for the carts by clicking <Select Measurement> on the y-axis and then selecting the desired velocities.
24. Since the carts move together, $v' = V'$. On the same graph, you should see velocity profiles where each picket fence goes through the right gate. For $v' = V'$, use maximum speed for the velocity profile that is closest to $t = 0$. on the graphs, as these are the speeds immediately before and after the collision.

25. Record v , v' and V' .

26. Complete the second row of the Observations table.

4. Discussion and Conclusion

Use equations (1) and (2) to complete the Results table. For collision 1: was momentum conserved; was kinetic energy conserved? For collision 2 was momentum conserved; was kinetic energy conserved? What types of collisions were collisions 1 and 2? What are two possible sources of error?

Table 1: Observations

	m (kg)	M (kg)	v (m/s)	v' (m/s)	V (m/s)	V' (m/s)
collision 1						
collision 2						
Date: Instructor's Name: Instructor's Signature:						

Table 2: Results

	$\vec{p}_{initial}$ (kg · m/s)	\vec{p}_{final} (kg · m/s)	$K_{initial}$ (J)	K_{final} (J)
collision 1				
collision 2				

Experiment 7: Rotation and Moment of Inertial

UNBC Department of Physics

August 25, 2014

1. Introduction

This experiment examines rotational motion and the moment of inertia. We will experimentally test the theoretical relationship between the torque applied to a wheel, the acceleration of the wheel, and the moment of inertia of the wheel using the method of graphical analysis.

Consider a thin cord that is wound around a rotating platform, with the hanging end of the cord used to suspend a mass, M , as shown in the diagram. To investigate the motion of the system, we need to specify the forces acting on the mass and the torques acting on the wheel.

The mass experiences a downward force, Mg , due to gravity, and an upward force, T , from the tension in the cord. The mass will accelerate downward with magnitude a according to Newton's second law,

$$Mg - T = Ma. \quad (1)$$

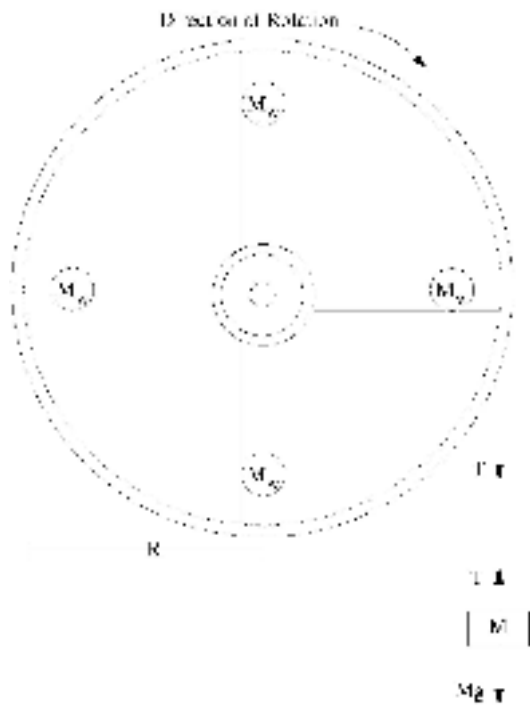
Notice that the tension, T , must be less than the force, Mg , if the mass is to accelerate downward.

Given that the mass accelerates downward, the wheel will rotate with angular acceleration, α . The tension in the cord acts at the edge of the spindle, and this produces a torque of magnitude $\tau = RT$ on the wheel, where R is the distance from the edge of the spindle to the centre of the axis of rotation.

Assuming that the torque due to friction is negligible, the magnitude of the angular acceleration, α , is given by

$$RT = I\alpha, \quad (2)$$

where I is the moment of inertia of the platform. The angular and linear accelerations



are related by

$$a = R\alpha, \quad (3)$$

because the acceleration of the hanging mass and the acceleration of the edge of the platform have the same magnitude.

Substituting Equation (3) into Equation (2), we get

$$RT = \frac{Ia}{R} \quad (4)$$

To show how tension, T , and acceleration, a , are related, we rearrange equation (1),

$$T = Mg - Ma. \quad (5)$$

Using equation(5) in equation (4) and rearranging gives (**show this**)

$$\frac{1}{a} = \frac{I}{gR^2} \frac{1}{M} + \frac{1}{g}.$$

This is the equation of a straight line $y = mx + b$, with $y = 1/a$, $x = 1/M$, and slope (not mass) $m = I/(gR^2)$

The mass M at the end of the cord is known, and the acceleration, a , of the mass is given in terms of the distance, h , through which it falls in a time, t , by

$$a = \frac{2h}{t^2}, \quad (6)$$

or

$$\frac{1}{a} = \frac{t^2}{2h}. \quad (7)$$

2. Apparatus

- rotating platform
- additional rotating mass
- stand and clamp
- string
- mass hanger (5 g) and mass set (2x20 g, 1x10 g, 1x5 g)
- stopwatch

3. Procedure

Determining the Moment of Inertia of the Platform

1. Spend a few moments familiarizing yourself with the apparatus and thinking about the best way to collect the data. The radius of the platform is $R = 0.20$ m in lab 8-227 and $R =$ in 8-229.
2. Start by selecting the position from which you will release the hanging mass. Measure and record the height h through which the hanging mass will descend after it is released. Also record the error in the height, δh , which is 0.5 mm.
3. For your first trial, let the hanging mass, M , consist of the hanger (5 g), with a 5 g mass added, for a total of 10 g. Record this value in Table 1.
4. Release the mass from rest and start the stopwatch at the same instant. Wait for the mass to reach the floor and stop the stopwatch at the instant that the mass comes into contact with the floor. Record your value for the total time, t_1 in Table 1.

5. Repeat four more trials for the same mass. Record the times of descent as t_2 , t_3 , t_4 , and t_5 .
6. Knowing that you made five trials $N = 5$, calculate and record the average time of fall, $\bar{t} = \frac{t_1+t_2+t_3+t_4+t_5}{5}$, the standard deviation of your time measurements, $\sigma_{s.d.} = \sqrt{\frac{(t_1-\bar{t})^2+(t_2-\bar{t})^2+(t_3-\bar{t})^2+(t_4-\bar{t})^2+(t_5-\bar{t})^2}{5-1}}$, and the error in \bar{t} , $\delta\bar{t} = \frac{\sigma_{s.d.}}{\sqrt{5}}$.
7. Use Equation (7) to calculate $\frac{1}{a} = \frac{\bar{t}^2}{2h}$. Calculate the error in $\frac{1}{a}$, $\delta\left(\frac{1}{a}\right) = \frac{1}{a}\sqrt{\left(\frac{\delta h}{h}\right)^2 + \left(\frac{2\delta\bar{t}}{\bar{t}}\right)^2}$.
8. Repeat steps 4 through 7. for masses of 20, 30, 40, 50 and 60 g (These values include the 5g of the hanger). Record your values in Table 1.
9. Use the data collected to plot a graph of $y = 1/a$ versus $x = 1/M$. Use the calculated values of $\delta(1/a)$ to draw vertical error bars. Determine the best line through your points (It must intersect at least two points and have an equal number of points below and above the line) and use the error bars to find the maximum (steepest) and minimum (shallowest) slopes.
10. Determine the value of the best slope, m_{best} , using two points you DID NOT plot, (x_1, y_1) and (x_2, y_2) and the formula: $m = \frac{rise}{run} = \frac{y_2-y_1}{x_2-x_1}$. Similarly find the values of m_{max} and m_{min} .
11. Find the error in the slope, δm , by calculating $m_{max} - m_{best}$ and $m_{best} - m_{min}$ and choosing the larger value.
12. Using slope $m = \frac{I_{platform}}{gR^2}$, find the moment of inertia of the platform, $I_{platform}$, and its error, $\delta I_{platform}$.

Find the Moment of Inertia of Added Mass

13. Adding mass to the platform changes the moment of inertia of the (platform plus mass) system to

$$I_{system} = I_{platform} + I_{mass} \quad (8)$$
14. Place additional mass on the platform. In lab 8-227, screw 4 additional masses (each added mass is 225 g) into the four 4th holes from the axle. In lab 8-229, attach the ring to the platform.
15. Repeat steps 2. through 12. to find I_{system} and δI_{system} .
16. Use equation (8) to find the moment of inertial I_{mass} of the added mass.

17. Compare I_{mass} to the directly measured value I_{theory} . In lab 8-227, $I_{\text{theory}} = M_{\text{added}}r^2$, where $r = 0.15\text{ m}$ is the location at which the mass is added. In lab 8-229, $I_{\text{theory}} = \frac{1}{2}M_{\text{added}}(r_{\text{inner}}^2 + r_{\text{outer}}^2)$, where r_{inner} and r_{outer} are the inner and outer radii of the ring (half the diameters).

4. Discussion and Conclusion

Comment on whether your experimental results agree with the theoretical predictions. Explain. Outline and discuss at least two possible sources of error which may have occurred during the collection of the data and how they could have been avoided.

Table 1: Determining the Moment of Inertia of the Platform

M (kg)						
$\frac{1}{M}$ (kg ⁻¹)						
t_1 (s)						
t_2 (s)						
t_3 (s)						
t_4 (s)						
t_5 (s)						
\bar{t} (s)						
$\sigma_{s.d.}$ (s)						
$\delta \bar{t}$ (s)						
$\frac{1}{a}$ (m ⁻¹ s ²)						
$\delta \left(\frac{1}{a} \right)$ (m ⁻¹ s ²)						

Date:

Instructor's Name:

Instructor's Signature:

Table 2: Determining the Moment of Inertia of the System

M (kg)						
$\frac{1}{M}$ (kg ⁻¹)						
t_1 (s)						
t_2 (s)						
t_3 (s)						
t_4 (s)						
t_5 (s)						
\bar{t} (s)						
$\sigma_{s.d.}$ (s)						
$\delta \bar{t}$ (s)						
$\frac{1}{a}$ (m ⁻¹ s ²)						
$\delta \left(\frac{1}{a} \right)$ (m ⁻¹ s ²)						

Date:

Instructor's Name:

Instructor's Signature:

Experiment 8: Standing Waves on a String

UNBC Department of Physics

August 25, 2014

1. Introduction

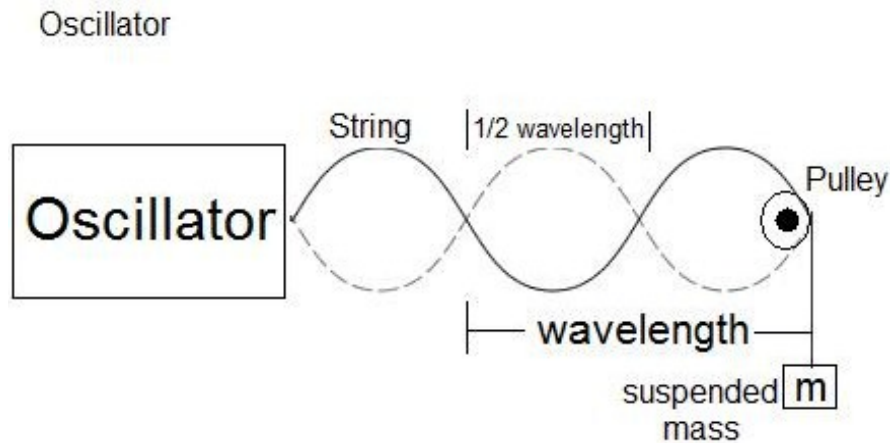
This experiment will be your first introduction to waves. You will generate standing waves in a taut string and use the method of dimensional analysis to construct a theoretical relationship between the wavelength λ of the standing waves and the tension T in the string.

Standing waves can be generated on a string by holding one end fixed and moving the other end up and down with simple harmonic motion of frequency f . A “transverse” wave travels down the string, is reflected at the fixed end, then travels back toward its starting point. We can think of the overall wave motion of the string as being due to the combination of two wavetrains having the same frequency but traveling in opposite directions. If the string has “just the right length”, the result will be a *standing wave* as shown in the diagram. A standing wave has a stable pattern with oscillating loops of length $\lambda/2$. The stationary points between the loops are called *nodes*, and the points midway between the nodes, where the amplitudes of oscillation are greatest, are called *antinodes*.

We can construct a relationship between the wavelength λ , frequency f , and tension T by using the method of dimensional analysis. In this method we relate physical quantities using empirical knowledge or intuitive insight, and we ensure that the dimensions (units) are the same on both sides of the equation. For example, we know from our experience with musical instruments that a heavier string vibrates at a different frequency than a lighter string. We therefore expect our relationship to involve the mass per unit length, ρ . (**Note, this is LINEAR density.**)

It is reasonable to try the following expression as a relationship between λ , ρ , f , and T :

$$\lambda = c\rho^\alpha f^\beta T^\gamma, \tag{1}$$



where c is a numerical constant having no dimensions, and the exponents α , β , and γ are to be found using dimensional analysis.

2. Apparatus

- string oscillator
- pulley and string
- tape measure
- mass hanger and mass Set (5x100g, 1x50g, 6x20g, 1x10g, 1x5g)

3. Advance Preparation

1. Using the method of dimensional analysis, determine the values of the exponents α , β , and γ in equation (1). Reconstruct equation (1) with these values of α , β , and γ .
2. Explain how you can test your equation using graphical analysis. What should be plotted on the “ y -axis”? What should be plotted on the “ x -axis”? Think carefully about this! Write your expression in the standard form $y = mx + b$. What is your expression for the slope m ?

4. Procedure

1. Spend a few moments familiarizing yourself with the various components of the apparatus shown in the diagram. A mechanical oscillator is used to generate the

standing waves. The tension in the string is adjusted by varying the mass hanging over the pulley.

2. Start by hanging the 50 g mass hanger from the pulley. Turn on the oscillator and **gently** add mass until a standing wave forms on the string.
3. “Fine-tune” the standing wave by further adjusting the mass until the “cleanest” possible standing wave results. The mass on the string is the value for the number of loops you have; record this value under M .
4. Measure the wavelength, λ , of this “cleanest” standing wave. (Note that the distance between nodes is $\lambda/2$.) Measure the width of one of the nodes, this is the error, $\delta\lambda$ in λ . (Notice that the nodes appear broadened instead of appearing at a sharp position.) Record your values of λ and $\delta\lambda$.
5. Resume adding mass until the number of loops changes.
6. Repeat steps 3 through 4 for smaller number of antinodes. For each trial, form and “fine-tune” a standing wave with a different wavelength (i.e. different number of oscillating loops). You can use your equation obtained in steps 1 and 2 of the **Advance Preparation** section to guide you. Collect data for as many different numbers of oscillating loops as you possibly can.
7. Measure the mass, M_{string} , and the length, L_{string} of the string provided by the lab instructor (**DO NOT disassemble your apparatus!**). Determine the density of the string, $\rho = \frac{M_{string}}{L_{string}}$ and its error: $\delta\rho = \rho \cdot \sqrt{\left(\frac{\delta M_{string}}{M_{string}}\right)^2 + \left(\frac{\delta L_{string}}{L_{string}}\right)^2}$.
8. Calculate and tabulate your values for what you determined y , δy , x and δx to be in step 2 of the the **Advance Preparation** section.
9. Plot a graph of y versus x . Use the calculated values of δx to draw horizontal error bars. Determine the best line through your points (It must intersect at least two points and have an equal number of points below and above the line) and use the error bars to find the maximum (steepest) and minimum (shallowest) slopes.
10. Determine the value of the best slope, m_{best} , using two points you DID NOT plot, (x_1, y_1) and (x_2, y_2) and the formula: $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$. Similarly find the values of m_{max} and m_{min} .
11. Find the error in the slope, δm , by calculating $m_{max} - m_{best}$ and $m_{best} - m_{min}$ and choosing the larger value.

12. Using the expression for slope that you found in step 2 of the **Advance Preparation** section and your results for the slope and its error, and given that the numerical value of c in Equation (1) is $c = 1$, calculate the value of the frequency, f and its error, δf .
13. Knowing that the wave velocity v is given by $v = f\lambda$, and taking the equation you found in step 1 of the the **Advance Preparation** section into account, find an expression for v in terms of T and ρ .

Hint: Look at the units in your formula for f and keep in mind that you are not pursuing a numerical answer.

5. Discussion and Conclusion

Comment on whether your experimental results agree with the theoretical predictions. Explain. Outline and discuss at least two possible sources of error which may have occurred during the collection of the data and how they could have been avoided.

Data Table: Newton's Second Law

Number of Antinodes	M (kg)	λ (m)	$\delta\lambda$ (m)

Date:

Instructor's Name:

Instructor's Signature:

Appendix A: Dimensional Analysis

UNBC Department of Physics

August 25, 2014

The method of dimensional analysis simply consists of making sure that the dimensions (units) on one side of an equation are the same as on the other side. For example, the force F required to keep an object moving in a circle is known to depend on the mass, m , of the object, its speed, v , and its radius, r , so we expect a relation of the form

$$F = cm^\alpha v^\beta r^\gamma, \quad (1)$$

where c is a numerical constant having no dimensions, and the exponents α , β , and γ will be found using dimensional analysis.

Writing the units for the quantities in equation (1), we have

$$\text{kg} \cdot \text{m} \cdot \text{s}^{-2} = \text{kg}^\alpha \cdot \left(\frac{\text{m}}{\text{s}}\right)^\beta \cdot \text{m}^\gamma.$$

Note: Force is measured in Newtons and a Newton is equal to $\text{kg} \cdot \text{m} / \text{s}^2$.

Making sure that (mass) is consistent on both sides of the equation tells us that $\alpha = 1$. In order that (time) has the power -2 on both sides we must have $\beta = +2$, since the unit for time on the right is already in the denominator. Finally, to get (length) to come out to the power of 1, we require that

$$1 = \beta + \gamma. \quad (2)$$

Using $\beta = 2$ in equation (2) gives

$$\begin{aligned} 1 &= 2 + \gamma \\ \gamma &= -1. \end{aligned}$$

Using these values of α , β , and γ , equation (1) becomes

$$F = cmv^2 r^{-1} = c \frac{mv^2}{r}.$$

Dimensional analysis does not tell us the value of c , but the description of circular motion shows that $c = 1$.

Appendix B: Error Analysis

UNBC Department of Physics

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1. Measurements and Experimental Errors

The word "error" normally means "mistake". In physical measurements, however, error means "uncertainty". We measure quantities like temperature, electric current, distance, time, speed, etc. ... using measuring devices like thermometers, ammeters, tape measures, watches etc. When a measurement of a certain quantity is performed, it is important to know how accurate or precise this measurement is. That is, a quantitative evaluation of how close we think this measurement is to the "true" value of the quantity must be established. An experimenter must learn how to assess the uncertainties or errors associated with a physical measurement so that he/she can convey a true picture of just how accurately a certain quantity has been measured. In general there are three possible sources or types of experimental errors: systematic, instrumental, and statistical. The following are guidelines which are commonly followed in defining and estimating these different forms of experimental errors.

1.1. Systematic Errors:

These errors occur repeatedly (systematically) every time a measurement is made and in general would be present to the same extent in each measurement. They arise because of miscalibration of a measuring device, ignoring some physical assumptions that should be taken into account when using an instrument, or simply misreading a measuring device. An example of a systematic error due to miscalibration is the recording of time at which an event occurs with a watch that is running late; every event will then be recorded late. Another example is reading an electric current with a meter that is not zeroed properly. If the needle of the meter points below zero when there is no current, then the reading of any current will be systematically less than the true current.

As an example of a systematic error due to ignoring some physical assumptions in a measurement, consider an experiment to measure the speed of sound in air by measuring the time it takes sound (from a gun shot, for example) to travel a known distance.

If there is a wind blowing in the same direction as the direction of travel of the sound then the measured speed of sound would be larger than the speed of sound in still air, no matter how many times we repeated the measurement. Clearly this systematic error is not due to any miscalibration or misreading of the measuring devices, but due to ignoring some physical factors that could influence the measurement.

An example of a systematic error due to misreading an instrument is parallax error. Parallax error is an optical error that occurs when a laboratory meter or similar instrument is read from one side rather than straight on. The needle of the meter would then appear to point to a reading on the side opposite to the side from which the meter is observed.

There is no general rule for the estimation of systematic errors. Each experimental situation has to be investigated individually for the possible sources and magnitudes of systematic errors. Care should be taken to reduce as much as possible the occurrence of systematic errors in a measurement. This could be done by making sure that all instruments are calibrated and read properly. The experimenter should be aware of all physical factors that could influence his/her measurement and attempt to compensate for or at least estimate the magnitude of such influences. A widely used technique for the elimination of systematic errors is to repeat the measurement in such a way that the systematic error adds to the measured quantity in one case and subtracts from it in another. In the example about measuring the speed of sound given above, if the measurement was repeated with the position of the source and detector of the sound interchanged, the wind speed would diminish the sound speed in this case, instead of adding to it. The effect of the wind speed will thus cancel when the average of the two measurements is taken.

1.2. Instrumental Uncertainties:

There is a limit to the precision with which a physical quantity can be measured with a given instrument. The precision of the instrument (which we call here the instrumental uncertainty) will depend on the physical principles on which the instrument works and how well the instrument is designed and built. Unless there is reason to believe otherwise, the precision of an instrument is taken to be the smallest readable scale division of the instrument. In this course we will report the instrumental uncertainty as one half of the smallest readable scale division. Thus if the smallest division on a meter stick is 1 mm, we report the length of an object whose edge falls between the 250 mm and 251 mm marks as (250.5 ± 0.5) mm. If the edge of the object falls much closer to the 250 mm mark than to the 251 mm mark, we may report its length as (250.0 ± 0.5) mm. Even though we might be able to estimate the reading of a meter to, let's say, 1/10 of the

smallest division we will still use one half of the smallest division as our instrumental uncertainty. Our assumption will be that had the manufacturer of the instrument thought that his/her device had a precision of 1/10 of the smallest division, then he/she would have told us so explicitly or else would have subdivided the reading into more divisions. For meters that give out a digital reading, we will take the instrumental uncertainty again to be one half of the least significant digit that the instrument reads. Thus if a digital meter reads 2.54 mA, we will take the instrumental error to be 0.005 mA, unless the specification sheet of the instrument says otherwise.

1.3. Statistical Errors:

Several measurements of the same physical quantity may give rise to a number of different results. This random or statistical difference between results arises from the many small and unpredictable disturbances that can influence a measurement. For example, suppose we perform an experiment to measure the time it takes a coin to fall from a height of two meters to the floor. If the instrumental uncertainty in the stop watch we use to measure the time is very small, say 1/100 second, repetitive measurement of the fall time will reveal different results. There are many unpredictable factors that influence the time measurement. For example, the reaction time of the person starting and stopping the watch might be slightly different on different trials; depending on how the coin is released, it might undergo a different number of flips as it falls each different time; the movement of air in the room could be different for the different tries, leading to a different air resistance. There could be many more such small influences on the measured fall time. These influences are random and hard to predict; sometimes they increase the measured fall time and sometimes they decrease it. Clearly the best estimate of the "true" fall time is obtained by taking a large number of measurements of the time, say $t_1, t_2, t_3, \dots, t_N$; and then calculating the average or arithmetic mean of these values.

The arithmetic mean, \bar{t} , is defined as

$$\bar{t} = \frac{t_1 + t_2 + t_3 + \dots + t_N}{N} = \frac{1}{N} \sum_{i=1}^N t_i, \quad (1)$$

where N is the total number of measurements.

To indicate the uncertainty or the spread in these measured values about the mean, a quantity called the standard deviation is generally computed. The standard deviation

of a set of measurements is defined as

$$\begin{aligned}\sigma_{\text{sd}} &= \sqrt{\frac{1}{N-1} (t_1 - \bar{t})^2 + (t_2 - \bar{t})^2 + (t_3 - \bar{t})^2 + \dots + (t_N - \bar{t})^2} \\ &= \sqrt{\frac{1}{N-1} \sum_{i=0}^N (\Delta t_i)^2},\end{aligned}\tag{2}$$

where $\Delta t_i = t_i - \bar{t}$ is the deviation and N is the number of measurements. The standard deviation is a measure of the spread in the measurements t_i . It is a good estimate of the error or uncertainty in each measurement t_i . The mean, however, is expected to have a smaller uncertainty than each individual measurement. We will show in the next section that the uncertainty in the mean is given by

$$\delta \bar{t} = \frac{\sigma_{\text{sd}}}{\sqrt{N}}.\tag{3}$$

According to the above equation, if four measurements are made, the uncertainty in the mean will be two times smaller than that of each individual measurement; if sixteen measurements are made the uncertainty in the mean will be four times smaller than that of an individual measurement, and so on; the larger the number of measurements, the smaller the uncertainty in the mean.

Note that a repeatedly measured quantity, the standard deviation of the measurement, and error in the mean, all have the same physical units.

Most scientific calculators have functions that calculate averages and standard deviations. Please take the time to learn how to use these statistical functions on your calculator. Some calculators have two functions for standard deviation, a sample standard deviation that uses $N - 1$ in the denominator of equation (2), and a population standard deviation that uses N in the denominator of equation (2). Make sure that you use the correct one; for TI (Texas Instruments) calculators, the correct function is `stdDev`.

Significance of the standard deviation: If a large number of measurements of an observable are made, and the average and standard deviation calculated, then 68% of the measurements will fall between the average minus the standard deviation and the average plus the standard deviation. For example, suppose the time it takes a coin to fall from a height of two meters was measured one hundred times, and the average was found to be 0.6389 s and the standard deviation 0.10 s. Then approximately 68% of those measurements will be between 0.5389 s (0.6389-0.10) and 0.7389 s (0.6389+0.10),

and of the remaining 32 measurements approximately 16 will be larger than 0.7389 s, and 16 will be less than 0.5389 s. In this example, the statistical uncertainty in the mean will be $0.010 \text{ s} \left(\frac{0.10}{\sqrt{100}} \right)$.

A comment about significant figures: The least significant figure used in reporting a result should be at the same decimal place as the uncertainty. Also, it is sufficient in most cases to report the uncertainty itself to one significant figure. Hence, in the example given above, the mean should be reported as $(0.64 \pm 0.01) \text{ s}$.

1.4. Combining Statistical and Instrumental Errors

In the following discussion, we will assume that care has been taken to eliminate all gross systematic errors, and that the remaining systematic errors are much smaller than the instrumental or statistical ones. Let IE stand for the instrumental error in a measurement and SE for the statistical error; then the total error, TE , is given by

$$TE = \sqrt{(IE)^2 + (SE)^2}. \quad (4)$$

In a good number of cases, either the IE or the SE will be dominant, and hence the total error TE will be approximately equal to the dominant error. If either error is less than one half of the other then it can be ignored. For example, if, in a length measurement, $SE = 1 \text{ mm}$ and $IE = 0.5 \text{ mm}$, then we can ignore the IE and say $TE = SE = 1 \text{ mm}$. If we had done the exact calculation (i.e., calculated TE from the above expression) we would have obtained $TE = 1.1 \text{ mm}$. Since it is sufficient to report the error to one significant figure, we see that the approximation we made in ignoring the IE is good enough.

Equation (3) suggests that the uncertainty in the mean gets smaller as the number of measurements is made larger. Can we make the uncertainty in the mean as small as we want by simply repeating the measurement the requisite number of times? The answer is no; repeating a measurement reduces the statistical error, not the total error. Once the statistical error is less than the instrumental error, repeating the measurement does not buy us anything, since, at that stage, the total error becomes dominated by the IE . In fact, you will encounter many situations in the lab where it will not be necessary to repeat the measurement at all because the IE error is the dominant one. These cases will usually be obvious. If you are not sure, then repeat the measurement three or four times. If you get the same result every time then the IE is the dominant error and there is no need to repeat the measurement further.

1.5. Propagation of Errors

Very often in experimental physics, a quantity gets "measured" indirectly by computing its value from other directly measured quantities. For example, the density of a given material might be determined by measuring the mass m and volume V of a specimen of the material, and then calculating the density as $\rho = \frac{m}{V}$. If the uncertainty in m is δm and the uncertainty in V is δV , what is the uncertainty in the density? We will not give a proof here, but the answer is given by

$$\delta\rho = \rho\sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta V}{V}\right)^2}.$$

More generally, if N independent quantities x_1, x_2, x_3, \dots are measured, and their errors x_1, x_2, x_3, \dots are determined, and if we wish to compute a function of these quantities,

$$y = y(x_1, x_2, x_3, \dots, x_N),$$

then the error in y is

$$\delta y = \sqrt{\left(\frac{\partial y}{\partial x_1}\delta x_1\right)^2 + \left(\frac{\partial y}{\partial x_2}\delta x_2\right)^2 + \left(\frac{\partial y}{\partial x_3}\delta x_3\right)^2 + \dots} = \sqrt{\sum_{i=1}^N \left(\frac{\partial y}{\partial x_i}\delta x_i\right)^2}. \quad (5)$$

Here, $\frac{\partial y}{\partial x_i}$ is the partial derivative of y with respect to x_i . Note that the x_i s are not repeated measurements of the same observable, they are measurements of independent variable. For example, x_1 could be a length, x_2 a time, x_3 a mass and so forth. It often makes the notation easier to call them x_i s rather than L, t, m and so forth.

Applying the above formula to the most common functions encountered in the lab give the following equations that can be used in error analysis.

1.5.1. Linear Functions

$$y = Ax, \quad (6)$$

where A is a constant (i.e., has no error associated with it). Then,

$$\delta y = A\delta x. \quad (7)$$

1.5.2. Addition and Subtraction

$$z = x \pm y \quad (8)$$

Then,

$$\delta z = \sqrt{(\delta x)^2 + (\delta y)^2}. \quad (9)$$

Clearly, if

$$z = Ax \pm By \quad (10)$$

where A and B are constants, then

$$\delta z = \sqrt{(A\delta x)^2 + (B\delta y)^2}. \quad (11)$$

1.5.3. Multiplication and Division

$$z = xy \quad \text{or} \quad z = \frac{x}{y} \quad (12)$$

Then,

$$\frac{\delta z}{z} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2}. \quad (13)$$

1.5.4. Exponents

$$z = x^n, \quad (14)$$

where n is a constant. Then,

$$\frac{\delta z}{z} = n \frac{\delta x}{x}. \quad (15)$$

For the more general case,

$$z = Ax^n y^m, \quad (16)$$

where A , n , and m are constants, equations (13) and (15) can be combined to give

$$\frac{\delta z}{z} = \sqrt{\left(\frac{n\delta x}{x}\right)^2 + \left(\frac{m\delta y}{y}\right)^2}. \quad (17)$$

Examples of the application equation (17):

$$\begin{array}{ll} z = Axy & \frac{\delta z}{z} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2} \\ z = Ax^2 & \frac{\delta z}{z} = 2 \frac{\delta x}{x} \\ z = A\sqrt{x} = Ax^{\frac{1}{2}} & \frac{\delta z}{z} = \frac{1}{2} \frac{\delta x}{x} \\ z = \frac{A}{x} = Ax^{-1} & \frac{\delta z}{z} = \frac{\delta x}{x}. \end{array} \quad (18)$$

1.5.5. Other Frequently Used Functions

$$\begin{array}{ll} y = A \sin(Bx) & \delta y = AB \cos(Bx) \delta x \\ y = e^x & \delta y = e^x \delta x \\ y = \ln x & \delta y = \frac{1}{x} \delta x \end{array} \quad (19)$$

1.5.6. Percent Difference

Calculating percent difference between an experimentally measured value and a theoretical value, or between an experimentally measured value and an accepted value, is often useful and appropriate.

$$\% \text{ difference} = \left| \frac{\text{theoretical value} - \text{experimental value}}{\text{theoretical value}} \right| \cdot 100\% \quad (20)$$