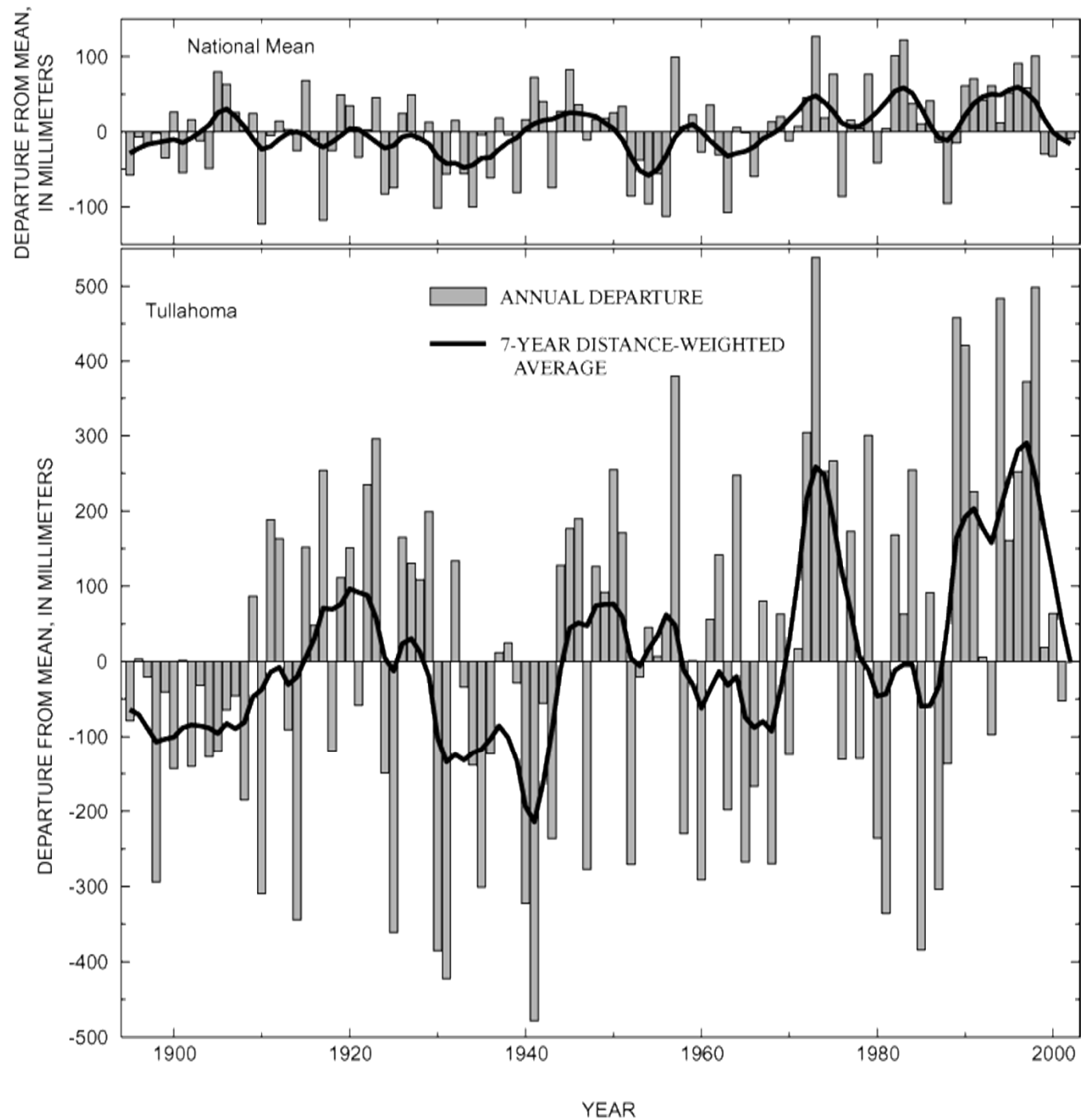


NRES_798_15_201501

Time series analysis

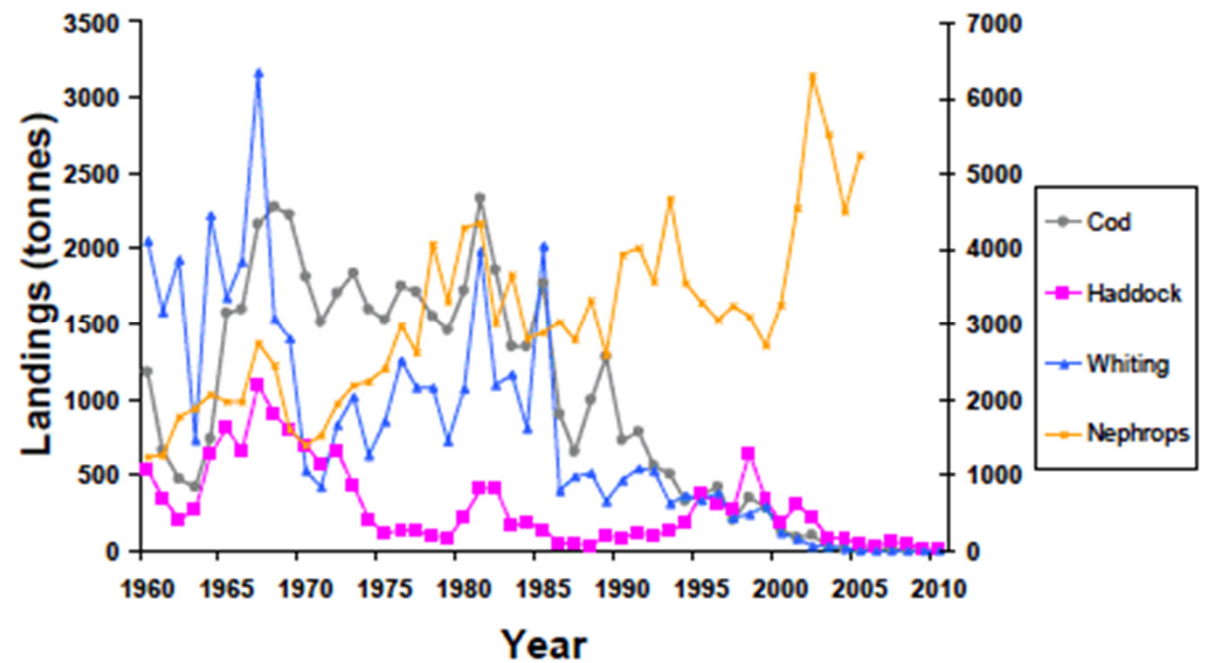
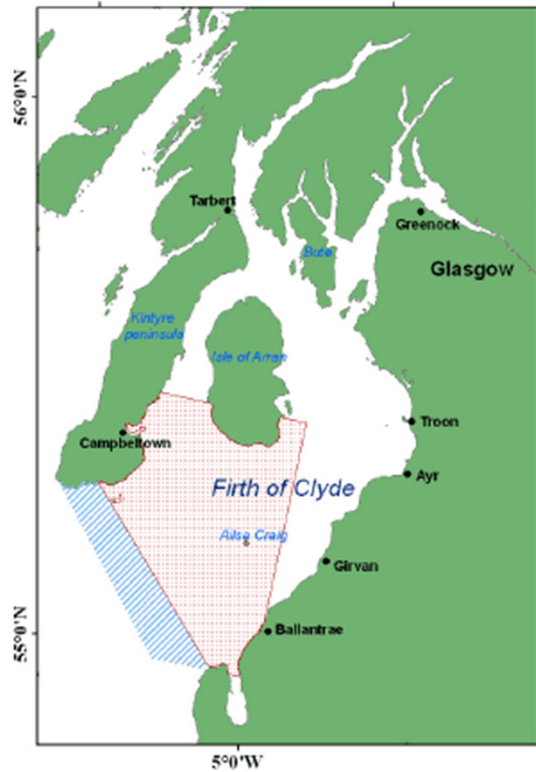
Time series analysis



Goals of time series analysis

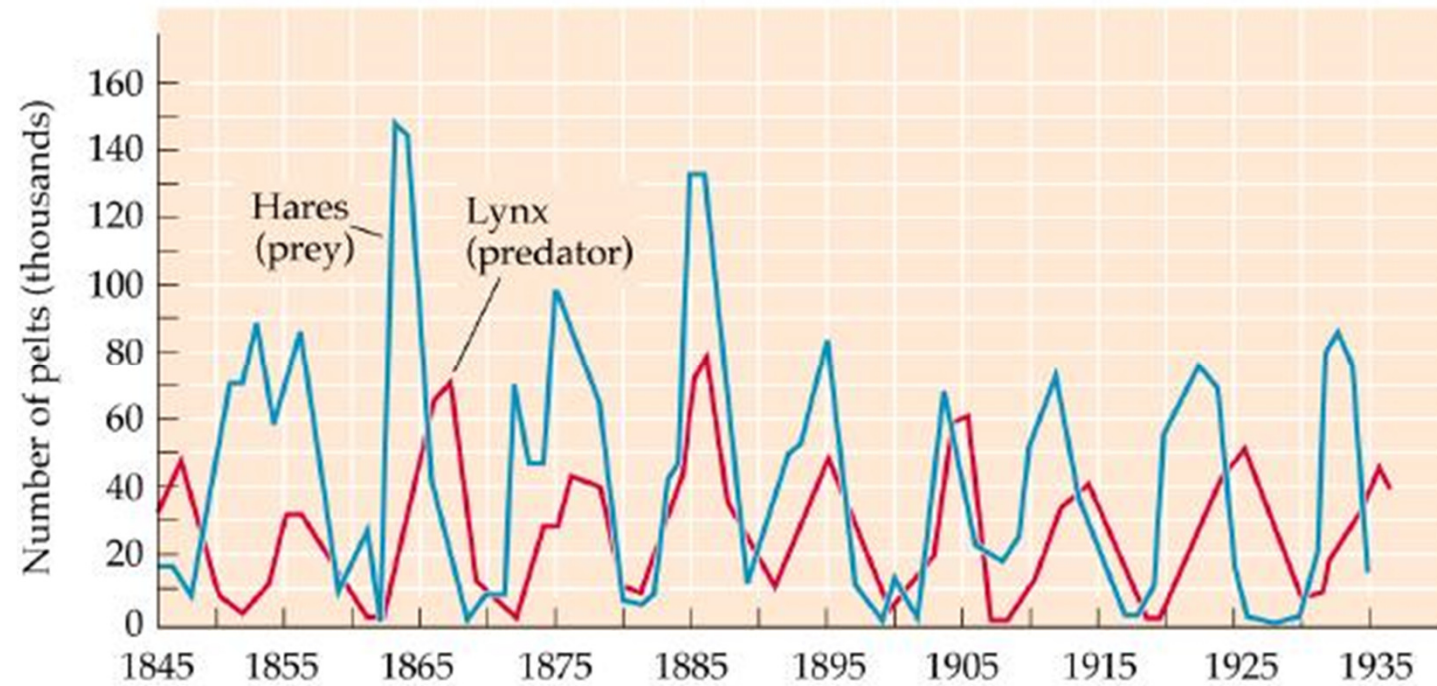
1. Identify the nature of the phenomenon represented by a sequence of observations
e.g. quantify strength of population regulation, stability of natural populations, identify population regulation mechanisms, determine if observations are cyclic
2. Forecasting (predicting future values of the time series variables).
e.g. conservation and stock management, assess population size and increasing or decreasing trends

Time series analysis



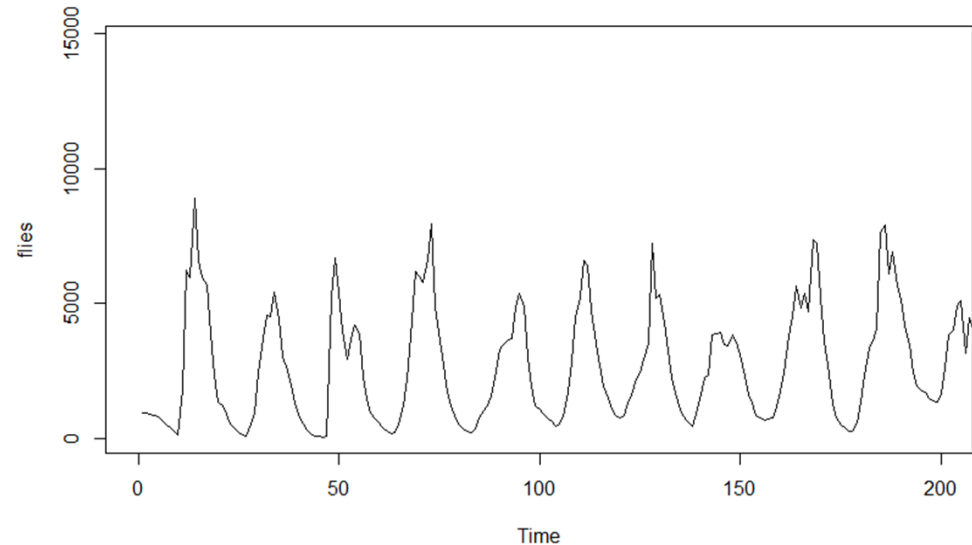
Scottish Marine and Freshwater
Science Volume 3 Number 3: Clyde
Ecosystem Review

Time series analysis



Time series

- Systematic pattern
- Random noise



- Time series analysis aims to filter out noise to make pattern more clear.
- Two basic classes of systematic components
 - Trend
 - Linear, nonlinear, does not repeat
 - Seasonal, cyclic
 - Repetition in systematic intervals

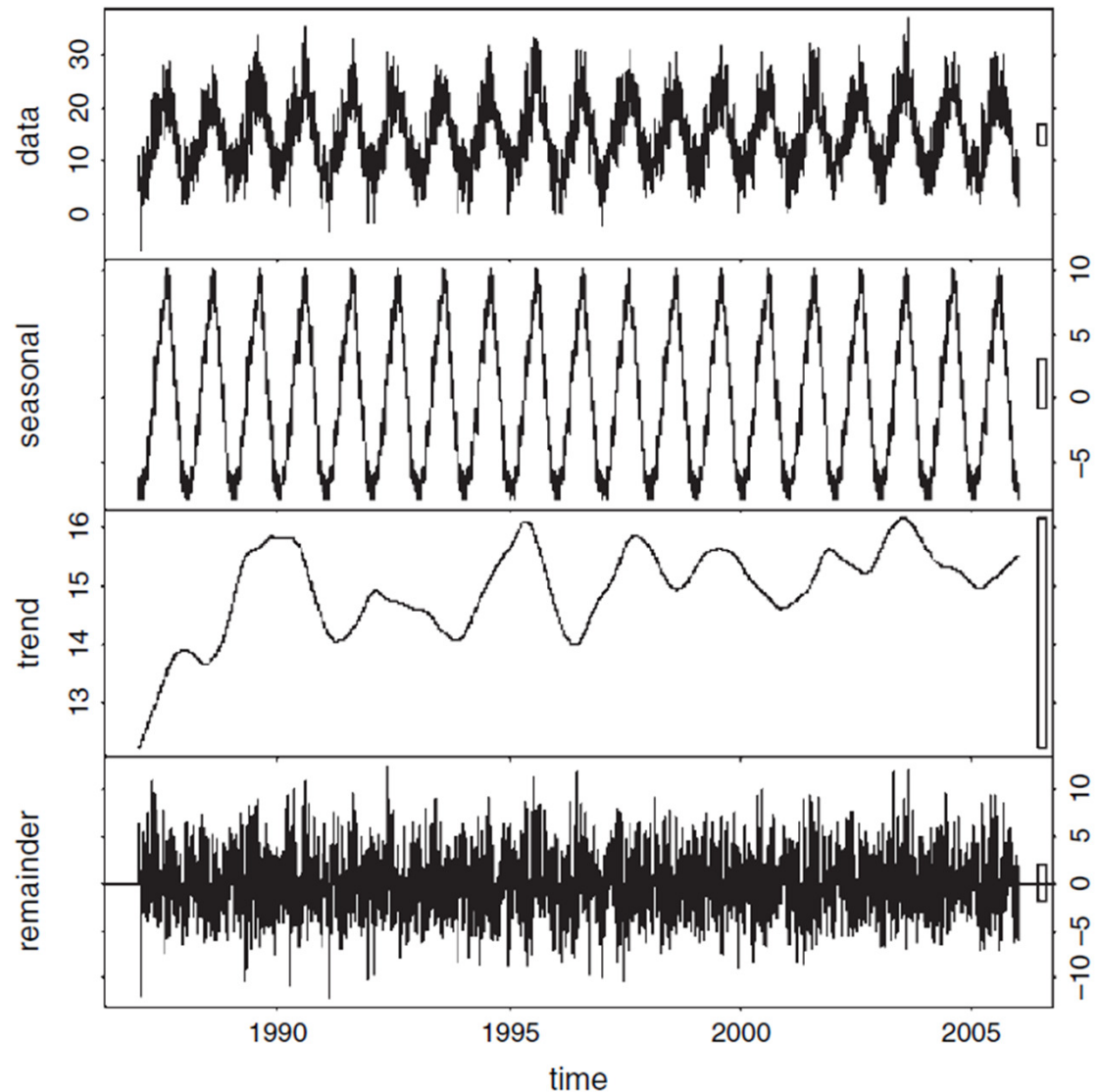
Classic decomposition

$$X_t = s_t + m_t + Y_t$$

S_t = seasonal component
(known period $d = 24$ (hourly), $d = 12$ (monthly))

M_2 = trend component (slowly
changing in time)

Y_t = random noise component
(might contain irregular cyclical
components of unknown
frequency + other noise).

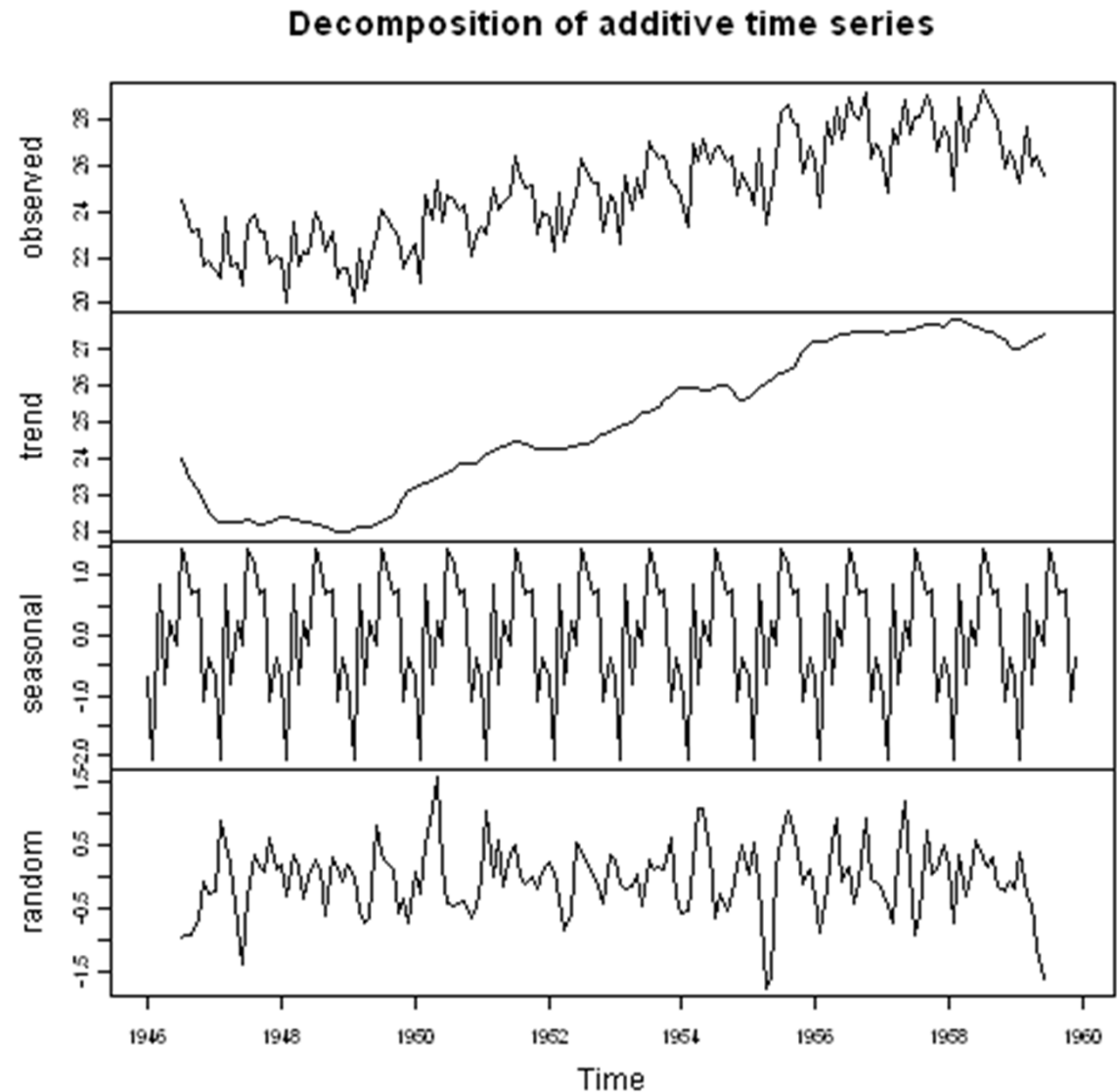


Classic decomposition

Births in New York

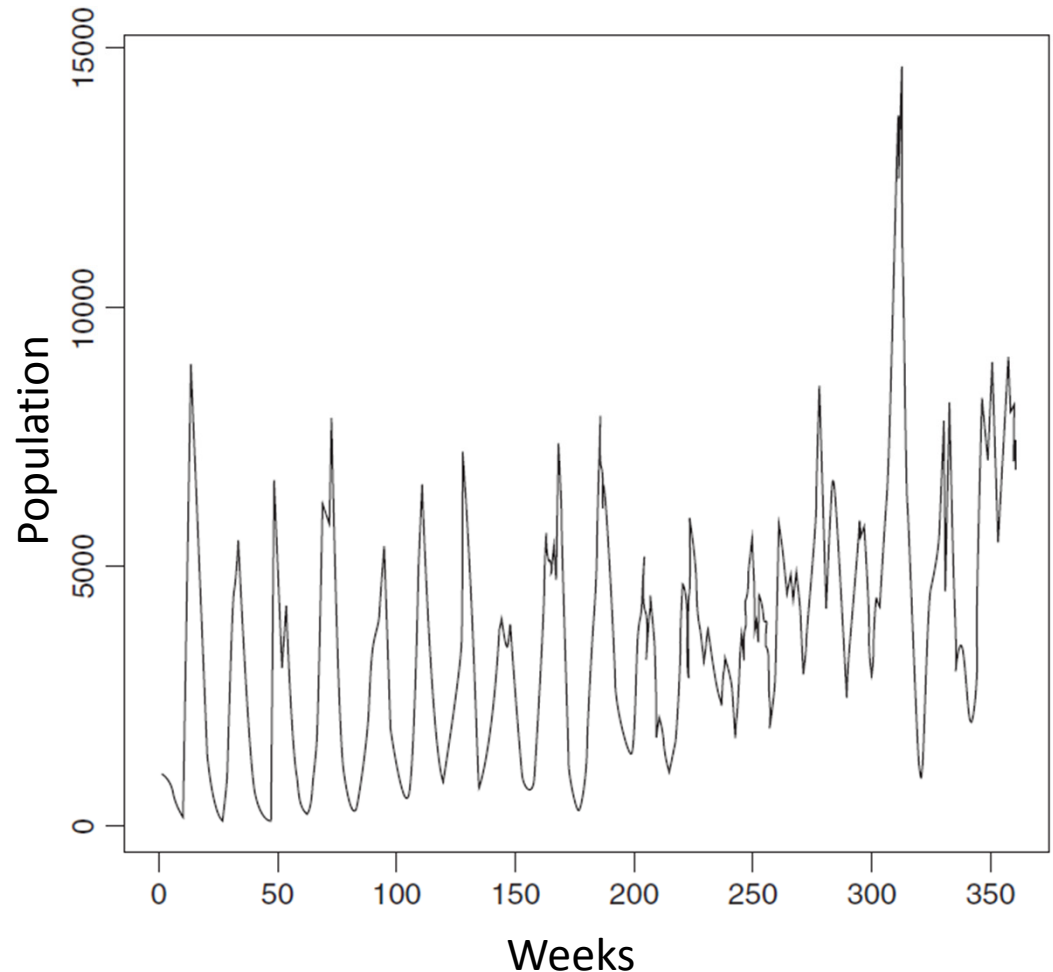
Seasonal decomposition of time series by loess

`stl(ts_object, "periodic")`



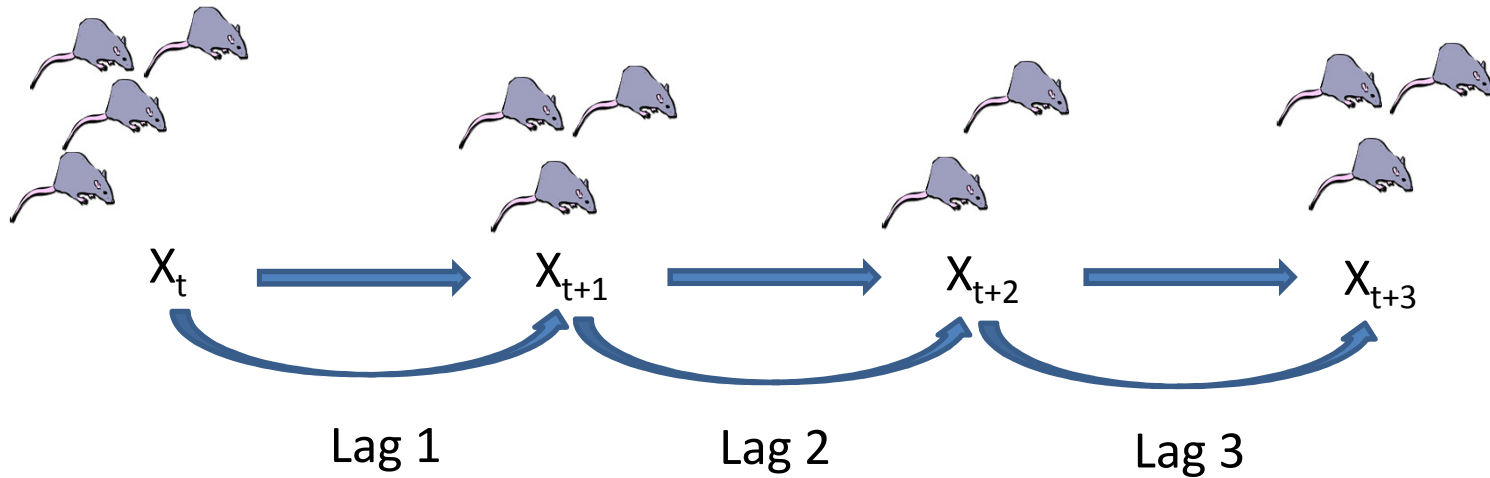
Identifying and modeling systematic patterns in time series

- Trend
 - Detrending
- Stationarity
 - Constant mean, variance, autocorrelation structure
 - Differencing
- Serial dependence
 - Autocorrelation
 - Moving average
 - Seasonal cycles
 - Spectral Analysis
 - Analysis of frequencies

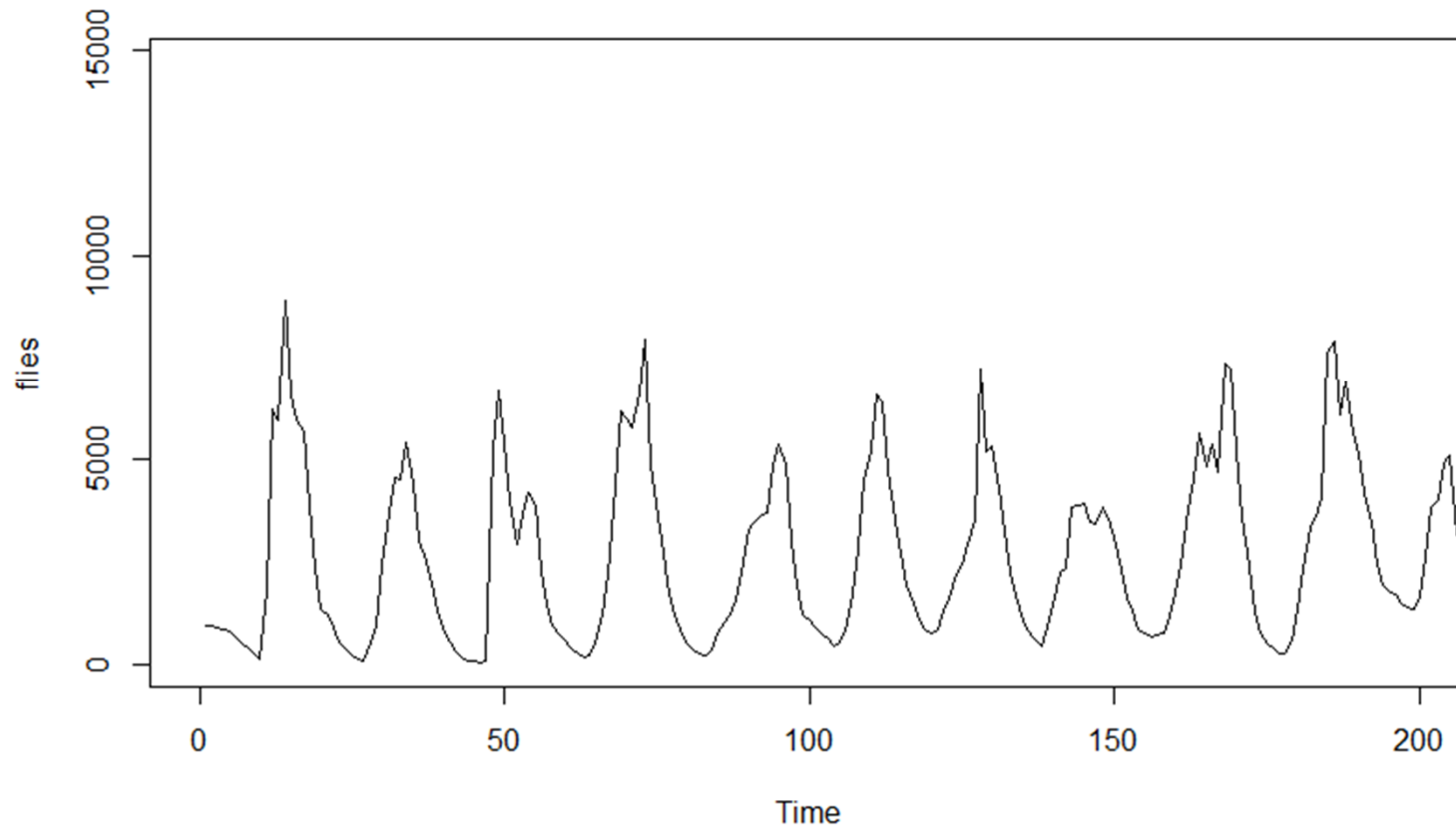


Autocorrelation

- How current population is related to previous population



Autocorrelation

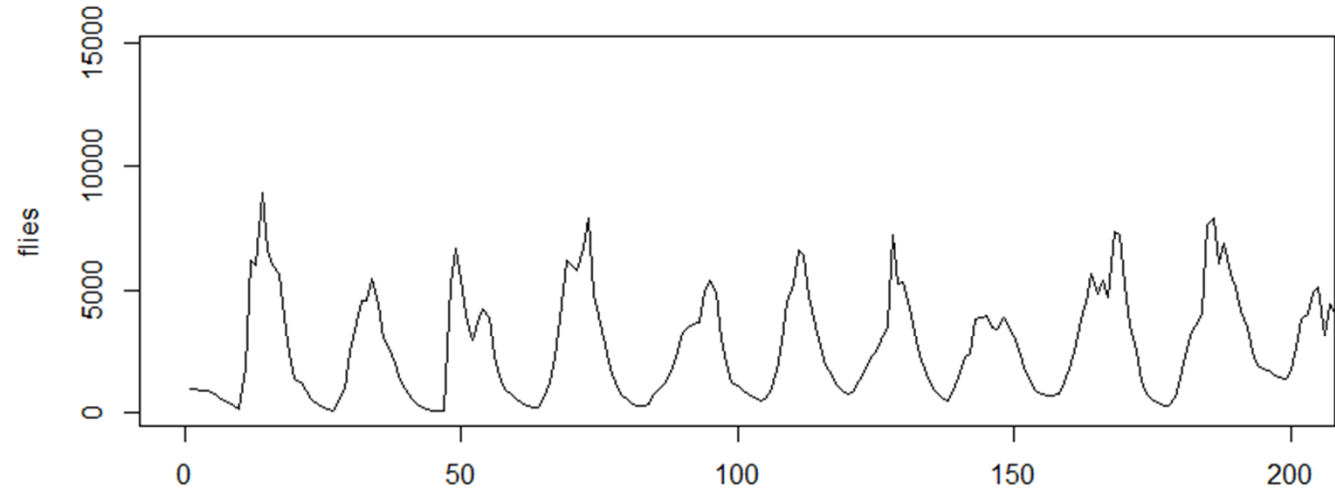


```
flies <- ts(blowfly$flies)  
plot(flies)
```

$$X_t = \sum_{i=1}^p \alpha_i X_{t-i} + \epsilon_t$$

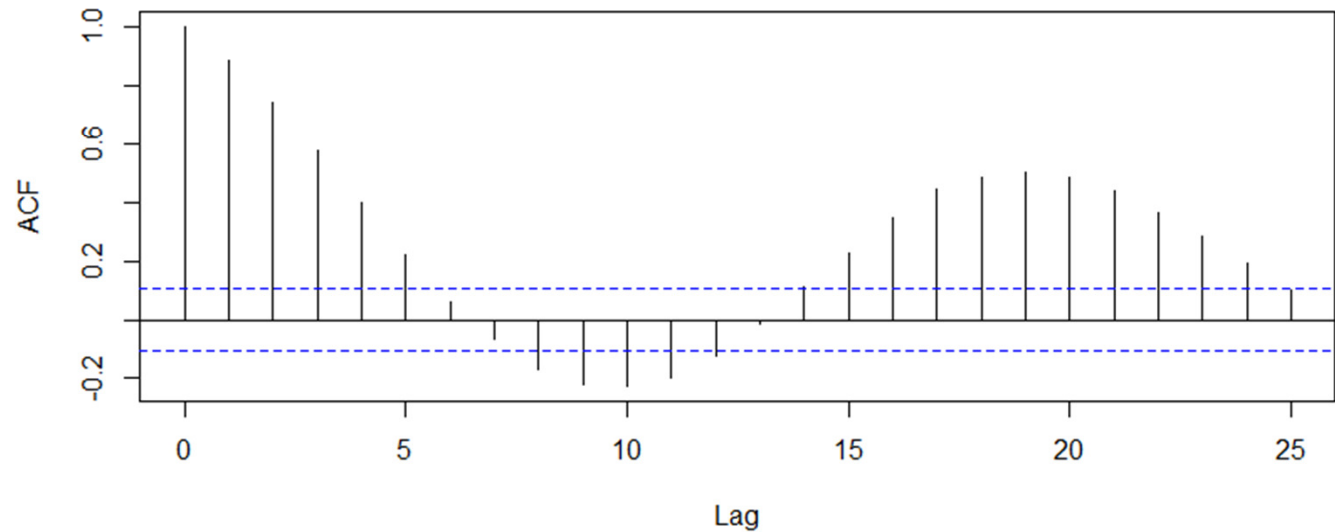
Autocorrelation

```
flies <- ts(blowfly$flies)  
plot(flies)
```



Autocorrelation function
estimation

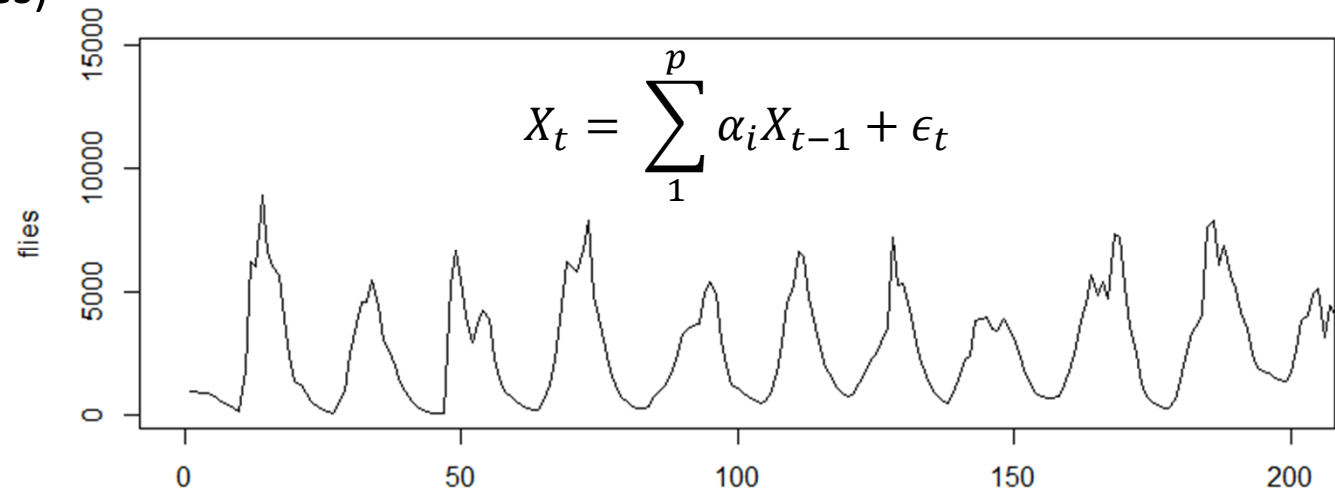
```
acf(flies, main="")
```



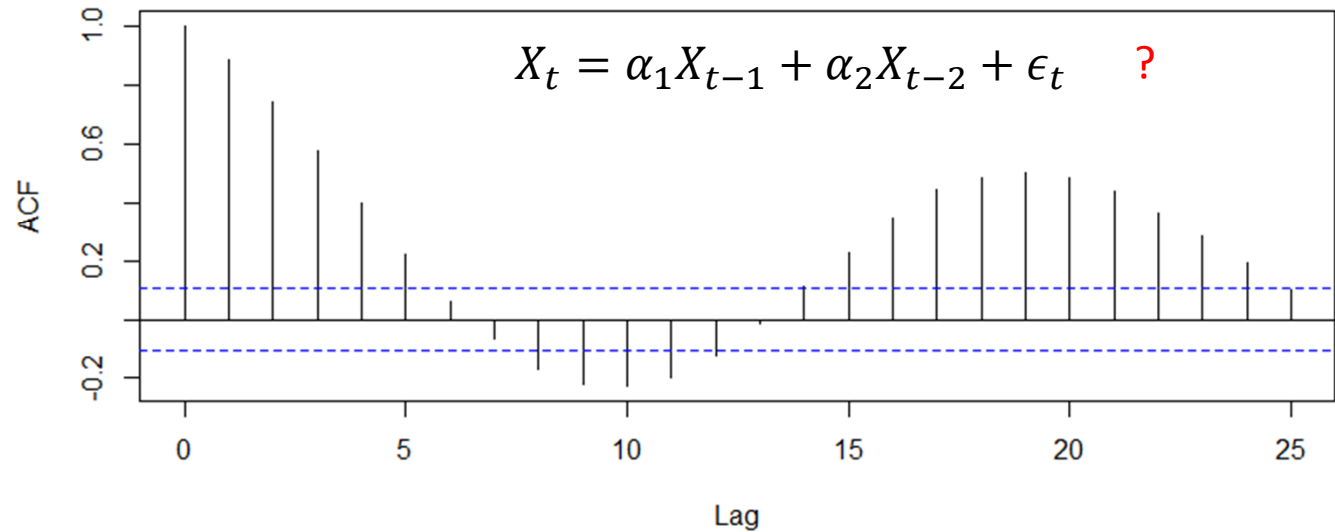
19 weeks

Autocorrelation

```
flies <- ts(blowfly$flies)  
plot(flies)
```



```
acf(flies, main="")
```

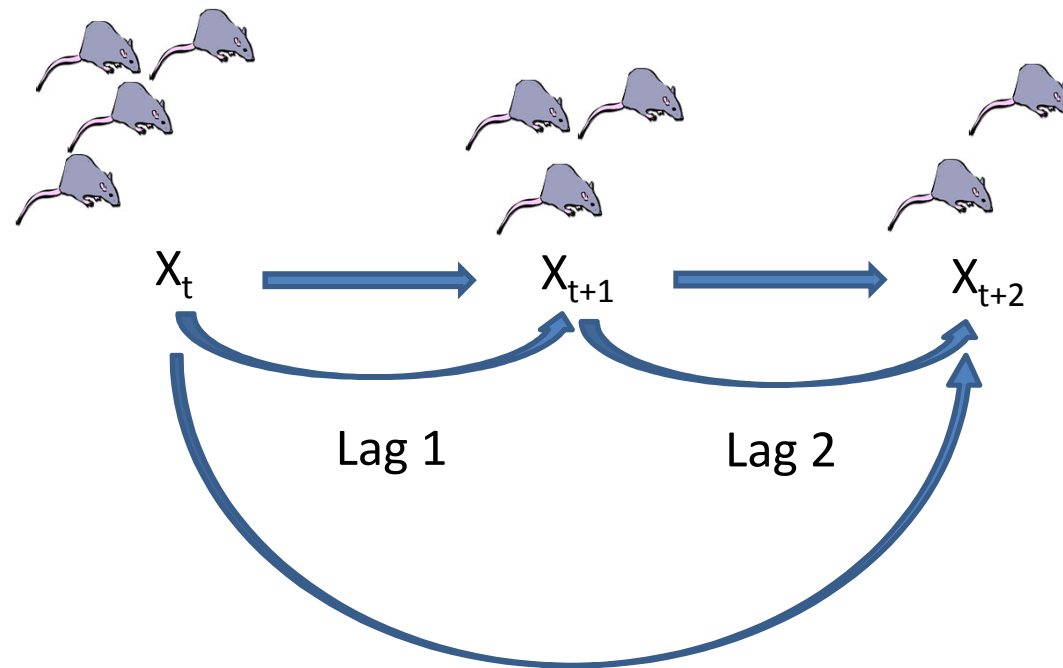


19 weeks

Partial autocorrelation

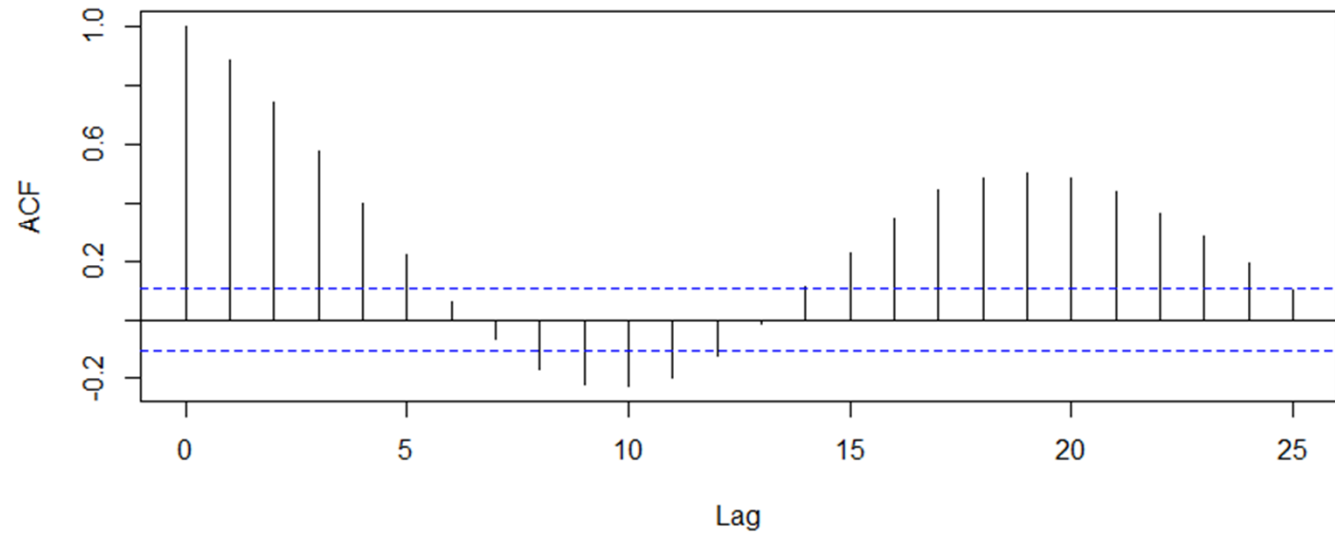
- Autocorrelation between X_t and X_{t+1} after removing linear dependence
 - i.e. once the correlation at lag 1 that “propagates” to lag 2 is removed.
- Valuable for understanding drivers of observed autocorrelation (i.e. identifying the appropriate lags in autocorrelation)
- Only a spike at lag 1, in partial autocorrelation, suggests that higher-order autocorrelations are effectively explained by the lag-1 autocorrelation

Partial autocorrelation

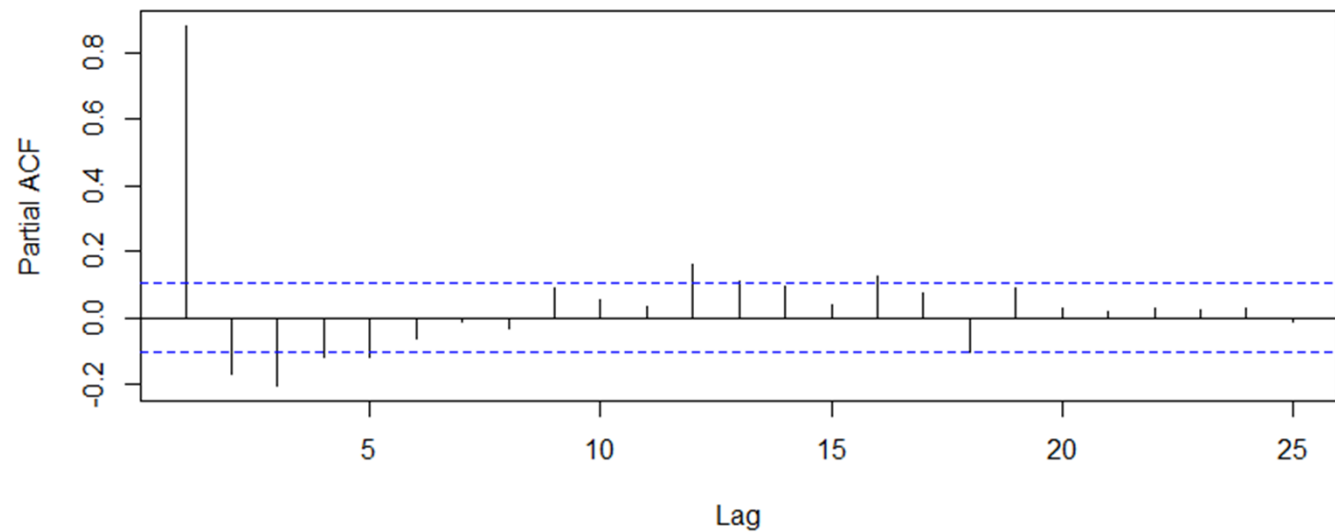


Partial autocorrelations

`acf(flies)`



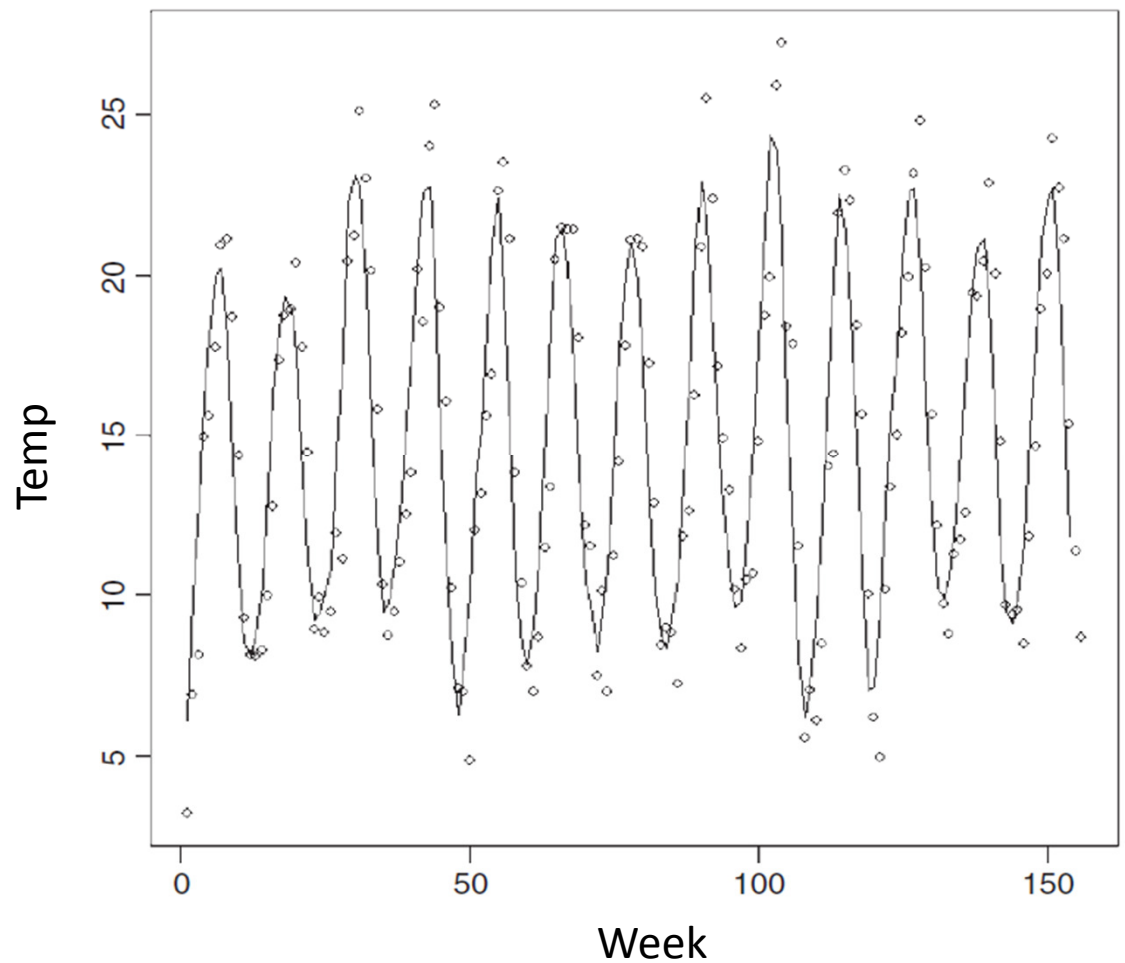
`acf(flies, type = "p")`



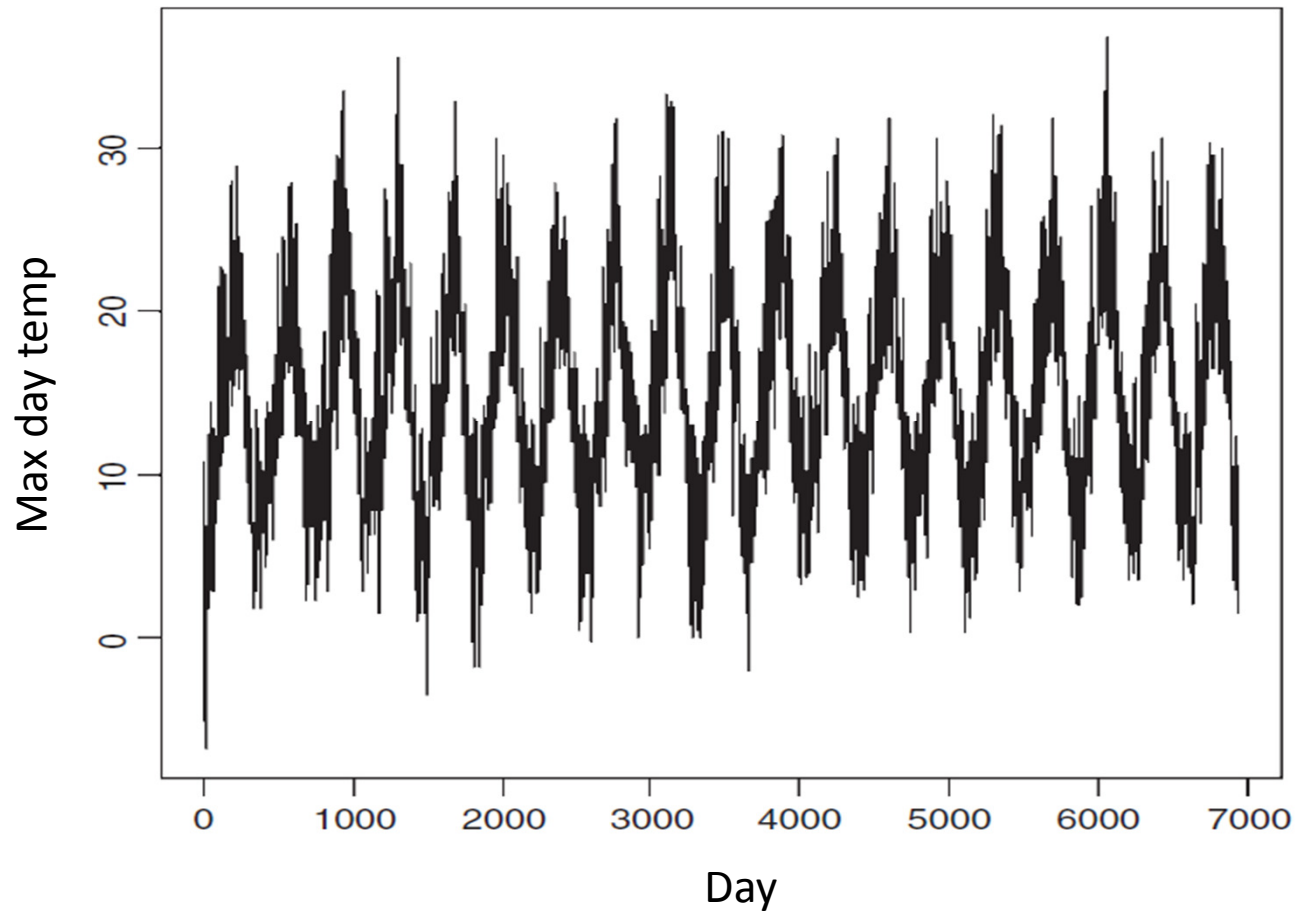
Moving average

$$Y^{\circ}_t = \frac{y_{t-1} + y_t + y_{t+1}}{3}$$

$$X_t = \sum_0^q \beta_j \epsilon_{t-j}$$

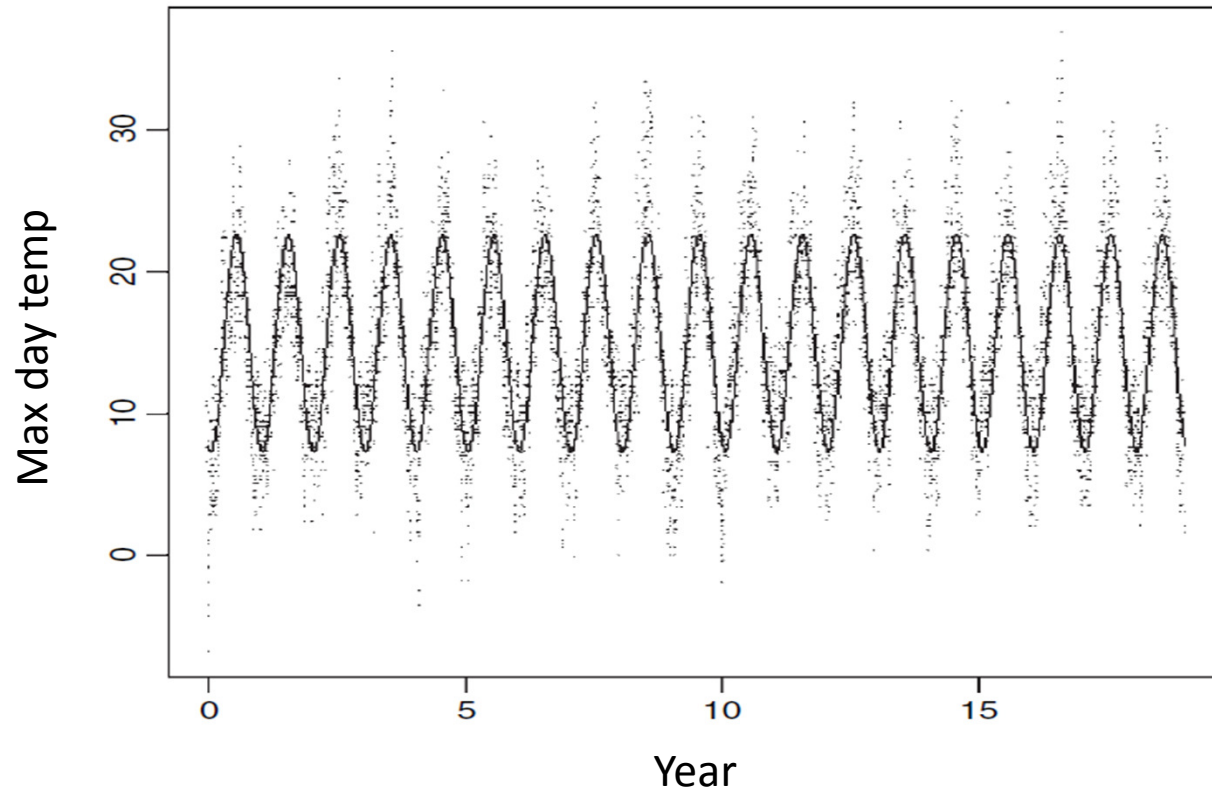


Seasonal Data



$$y = \alpha + \beta \sin(2\pi t) + \gamma \cos(2\pi t) + \varepsilon$$

Seasonal cycles



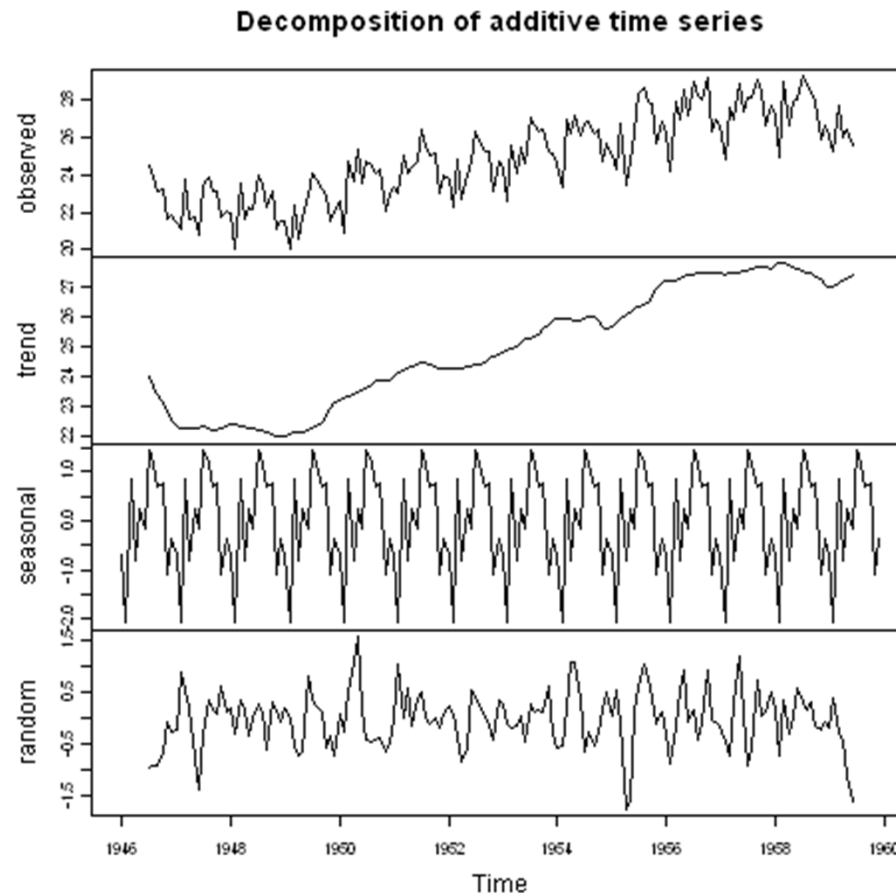
Time scaled so season length is 1 ($t_d/365$)

Fit linear model (after de-trending)

$$y = \alpha + \beta \sin(2\pi t) + \gamma \cos(2\pi t) + \varepsilon$$

Decomposition

- Seasonal decomposition of time series
- Data, seasonal, trend, remainder (random)
- `stl(ts_object, "periodic")`



Time Series Models

- Autoregressive (AR)

$$X_t = \sum_1^p \alpha_i X_{t-1} + \epsilon_t$$

- Moving average (MA)

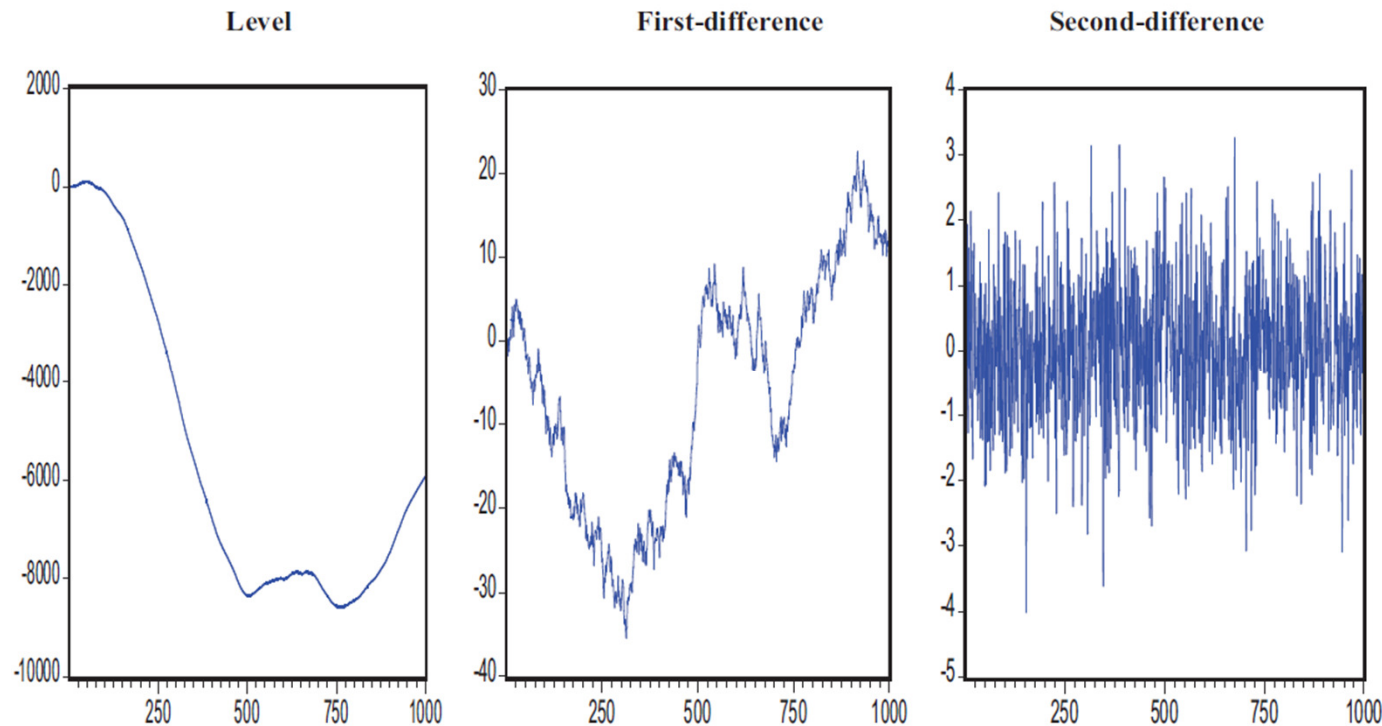
$$X_t = \sum_0^q \beta_j \epsilon_{t-j}$$

- Autoregressive moving average (ARIMA)
 - Box-Jenkins (Box and Jenkins 1976)

$$X_t = \sum_1^p \alpha_i X_{t-i} + \sum_0^q \beta_j \epsilon_{t-j}$$

Autoregressive moving average (ARIMA)

- Model selection
 - Confirm data is stationary (graphically)
 - Use differencing to achieve stationarity
 - i.e. identify d (difference order)



Autoregressive moving average (ARIMA)

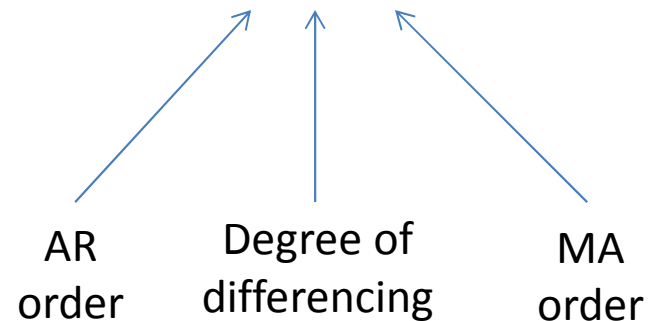
- Model selection (3 steps)
 - Confirm data is stationary (graphically)
 - Use differencing to achieve stationarity
 - i.e. identify d (difference order)
 - Identify if a seasonal component is necessary
 - Identify seasonality: data plot, acf plot
 - Include seasonal autoregressive term (SARIMA)
 - Identify the ARMA order (p,q)

- ARIMA model selection (3 steps)
 - Confirm data is stationary (graphically)
 - Identify if a seasonal component is necessary
 - Identify the ARMA order (p,q)
 - Use acf and pacf plots
 - Shape of acf function (art of interpretation)
 - Exponential decay to zero
 - » Autoregressive model (identify p using pacf)
 - Damped oscillations decaying (exponential) to zero
 - » Autoregressive model
 - One or more spikes, the rest is zero
 - » Moving average model, order q identified by where autocorrelation becomes zero
 - Exponential decay starting after a few lags
 - » Mixed autoregressive and moving average model
 - Identify potential min and max order for p and q and test all model combinations
 - Process focused ecological time series models will tend to be autoregressive.

Autoregressive moving average (ARMA)

- ARIMA in R

- Function `arima(ts_object, order=c(p,d,q))`



`arima(ts_object, order=c(2,1,0))`

`arima(ts_object, order=c(2,1,1))`

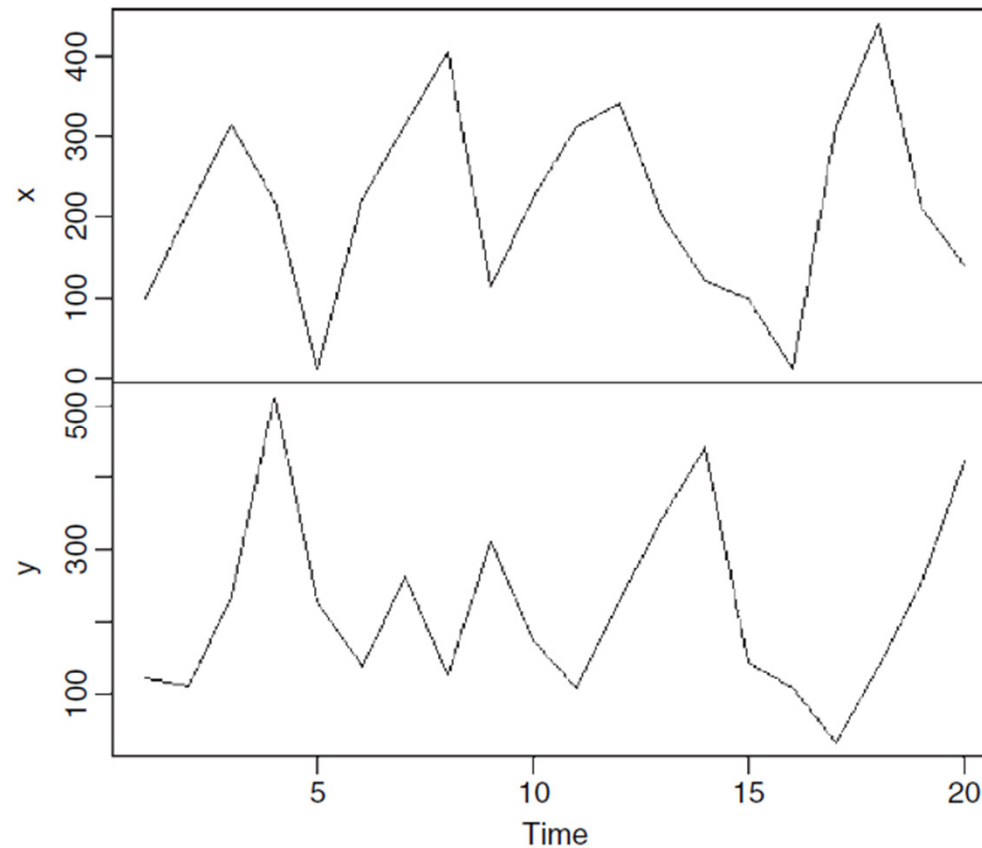
`arima(ts_object, order=c(1,1,1))`

`arima(ts_object, order=c(0,1,2))`

`AIC(m1,m2,m3,m4)`

Multiple time series

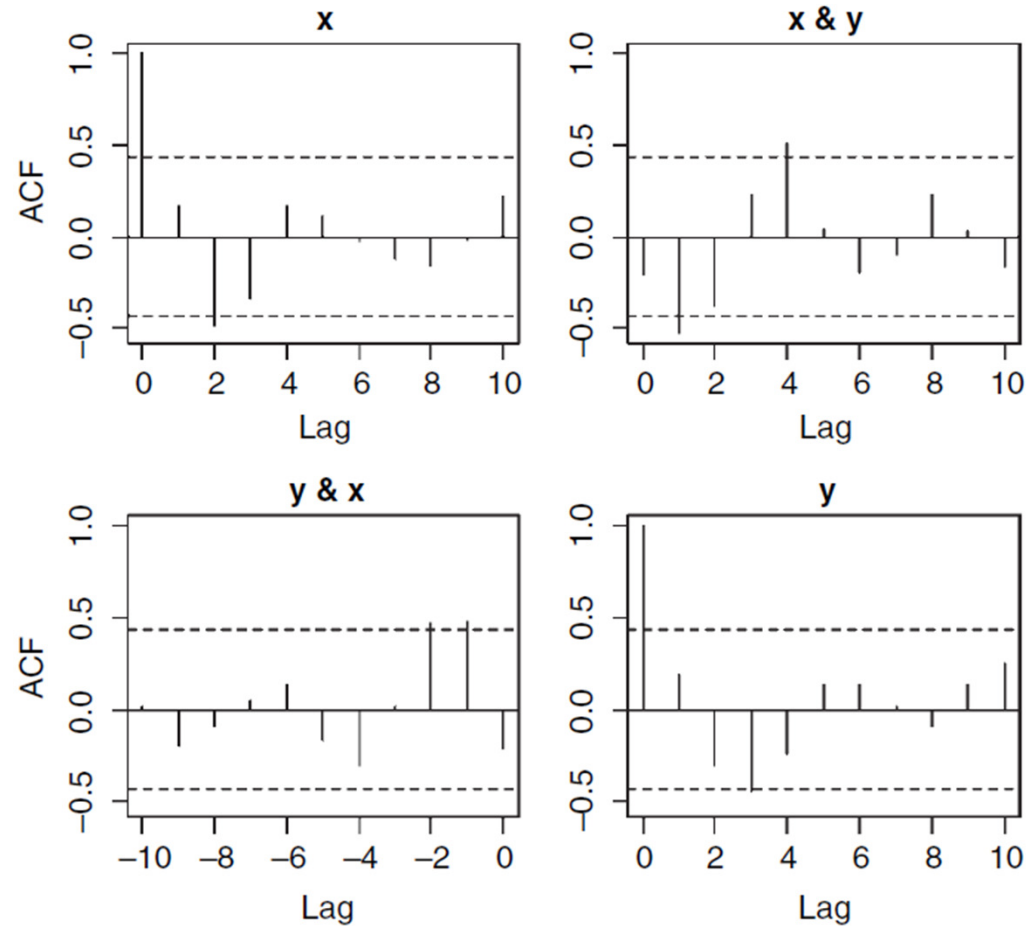
- Correlations between two time series



`acf(cbind(x,y,))`

Multiple time series

- Correlations between two time series



`acf(cbind(x,y,))`

R time series

- `ts(data)` : function to create time series object
- `acf(ts_data)` : autocorrelation function
- `pacv(ts_data)`: partial autocorrelation function
- `stl(ts_data,"periodic")` : Seasonal decomposition of time series
- `Arima(ts_data,order=c(p,d,q))` : arima time model