BIOL 410 Population and Community Ecology

Competition

Lotka-Volterra models

Competition

- Intraspecific competition (within species)
- Interspecific competition (between species)
- Negative interaction (-/-) between two or more species within the same trophic level
- Depresses population growth rate
  - Exploitation competition
  - Pre-emptive competition
  - Interference competition
    - Allelopathy

Lotka-Volterra Interspecific Competition Model

- Model involving two competing species – 1,2
- Each species grows according to a logistic growth model
  - own population size (N_1 and N_2)
  - intrinsic growth rates (r_1 & r_2)
  - carrying capacity (K_1 & K_2)

$$\frac{dN_1}{dt} = r_1 N_1 \left(\frac{K_1 - N_1}{K_1}\right)$$

$$\frac{dN_2}{dt} = r_2\ N_2 \left(\frac{K_2 - N_2}{K_2}\right)$$

Lotka-Volterra Interspecific Competition Model

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$$\frac{dN_1}{dt} = r_1 N_1 \left(\frac{K_1 - N_1 - N_2}{K_1} \right)$$

- Population 1 focus
- Assumes individual of pop 2. equivalent to individual of pop. 1
- Assumes logistic relationship

Lotka-Volterra models

- Population growth rate of species 1 will be negatively effected by the presence of species 2
- Competitors are not equal (equivalent) in their use of resources, therefor include a correction coefficient
  - Competition coefficient: the "efficiency" parameter

$$\frac{dN_1}{dt} = r_1 N_1 \left(\frac{K_1 - N_1 - \alpha N_2}{K_1} \right)$$

- α per capita effect of species 2 on (resources) growth rate of species 1

$$\frac{dN_2}{dt} = r_2 N_2 \left(\frac{K_2 - N_2 - \beta N_1}{K_2} \right)$$

- β per capita effect of species 1 on growth rate of species 2

Interspecific competition

- Problem: how does change in the growth of N2 influenced the growth of N_1, given r_1, r_2, k_1, k_2, α, β?

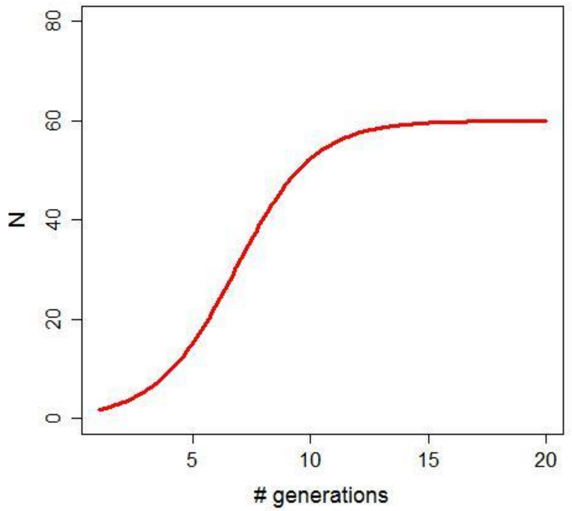
$$\frac{dN_1}{dt} = r_1 N_1 \left(\frac{K_1 - N_1 - \alpha N_2}{K_1} \right)$$

$$\frac{dN_2}{dt} = r_2 N_2 \left(\frac{K_2 - N_2 - \beta N_1}{K_2} \right)$$

- Paired differential equations have combined solutions

Interspecific competition

- Solving or competition
  - A simple, but complicated solution:
    - Evaluate each population at equilibrium
      - $dN_1/dt = 0$, $dN_2/dt = 0$



Combination of N_1 and N_2

$$\frac{dN_1}{dt} = r_1 N_1 \left(\frac{K_1 - (N_1 - \alpha N_2)}{K_1} \right)$$

Interspecific competition

- Solve for an equilibrium solution

$$0 = r_1 N_1 \left(\frac{K_1 - N_1 - \alpha N_2}{K_1} \right)$$

$$0 = r_2 N_2 \left(\frac{K_2 - N_2 - \beta N_1}{K_2} \right)$$

$$0 = K_1 - N_1 - \alpha N_2$$

$$0 = K_2 - N_2 - \beta N_1$$

$$\widehat{N_1} = K_1 - \alpha N_2$$

$$\widehat{N_2} = K_2 - \beta N_1$$

Equilibrium Population Values

To look at the effects entirely in terms of one species, substitute in the equilibrium N for the other species into it's equation.

$$\widehat{N}_1 = K_1 - \alpha N_2 \qquad \widehat{N}_2 = K_2 - \beta N_1$$

$$\widehat{\mathrm{N}}_1 = \mathrm{K}_1 - \alpha(\mathbf{K_2} - \boldsymbol{\beta}\mathbf{N_1})$$

$$\widehat{\mathrm{N}}_1 = \frac{\mathrm{K}_1 - \alpha \mathrm{K}_2}{1 - \alpha\beta}$$

Equilibrium Population Values

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$$\hat{N}_1 = K_1 - \alpha(\mathbf{K_2} - \boldsymbol{\beta}\mathbf{N_1}) \qquad \hat{\mathbf{N}}_\mathbf{2} = \mathbf{K_2} - \boldsymbol{\beta}(K_1 - \alpha N_2)$$

$$\hat{\boldsymbol{N}}_\mathbf{1} = \frac{\boldsymbol{K_1 - \alpha K_2}}{\boldsymbol{1 - \alpha\beta}} \qquad \hat{N}_2 = \frac{\boldsymbol{K_2 - \beta K_1}}{\boldsymbol{1 - \alpha\beta}}$$

For both species to have stable co-existence, the denominator in both equilibria equations ($\boldsymbol{1 - \alpha\beta}$) must be greater than zero.

That means the product $\boldsymbol{\alpha\beta}$ must be less than 1

Interspecific competition

Equilibrium solution (graphical)

Step #1: $0 = K_1 - N_1 - \alpha N_2$

Step #2: algebra to get the dN/dt = 0 isocline

$$N_1 = K_1 - \alpha N_2 \qquad N_2 = \frac{K_1 - N_1}{\alpha}$$

Step #3: calculate the two extremes to dN/dt =0:

- Solution #1: if species 1 wins and species 2 goes extinct

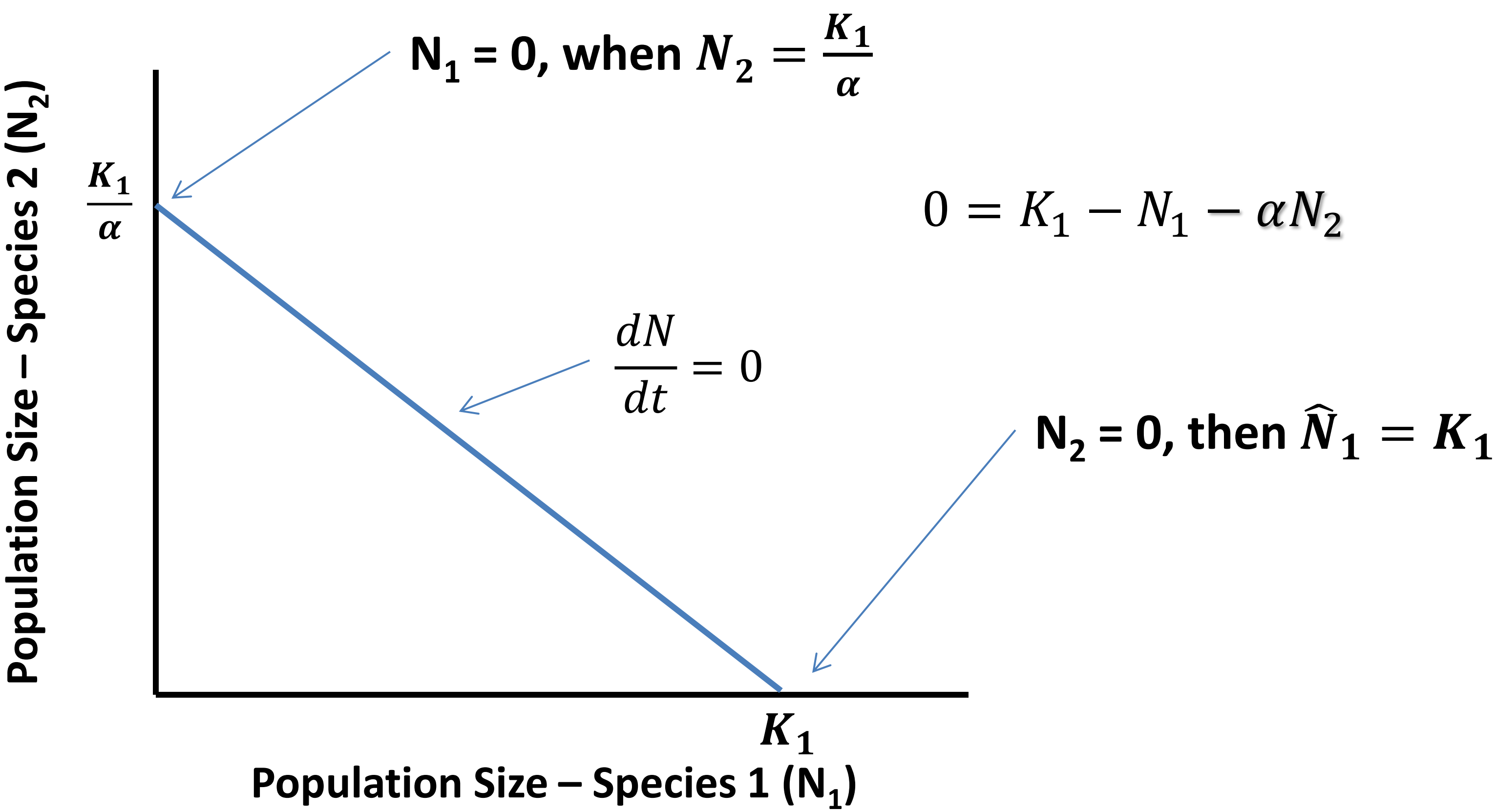
$$N_1 = K_1$$

- Solution #2: if species 2 wins and species 1 goes extinct

$$N_2 = \frac{K_1}{\alpha}$$

Isoclines and State-Space Graphs

- Plot equilibrium lines for each species as zero isoclines – relative population sizes where $dN_x/dt = 0$



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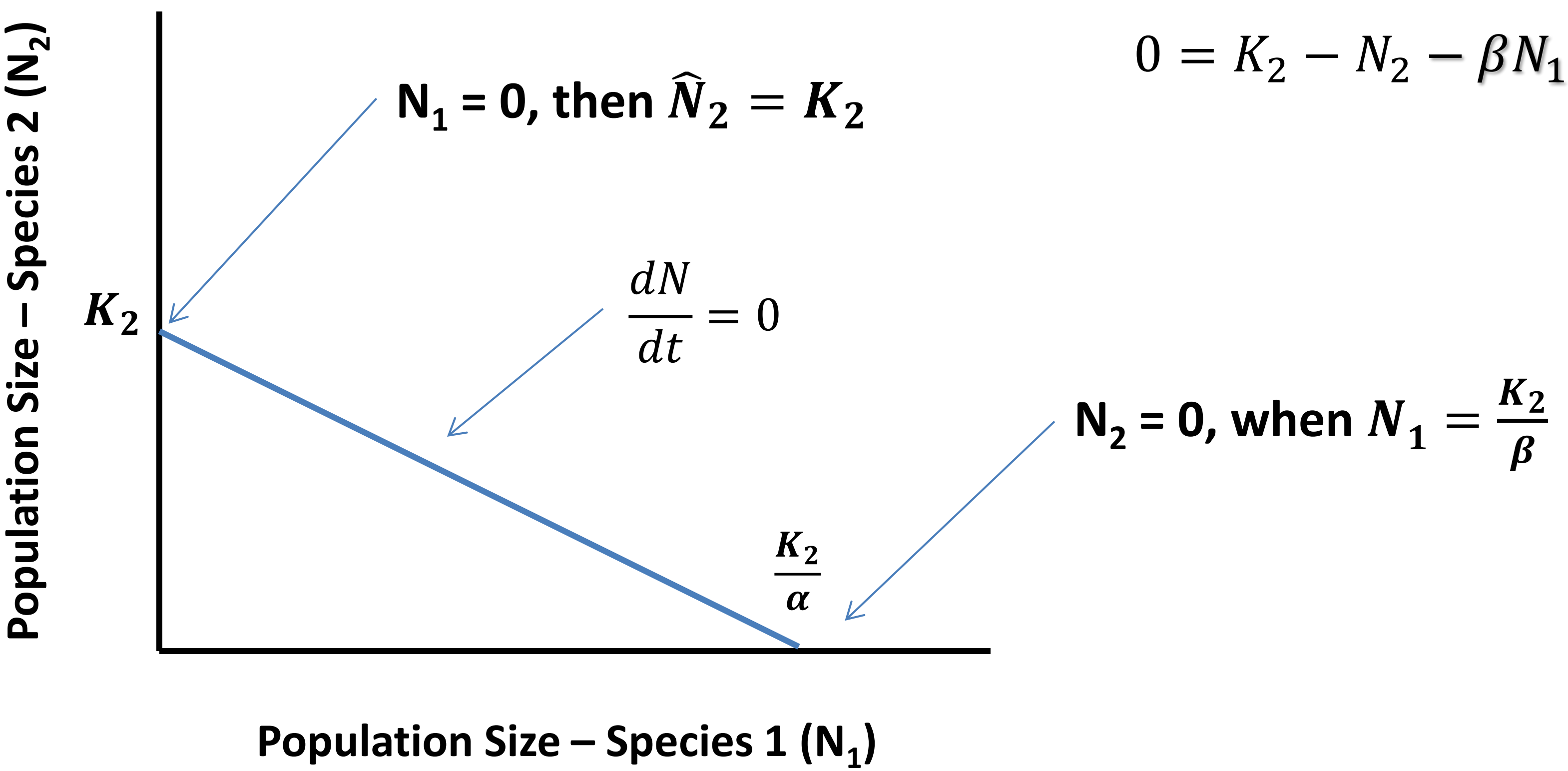
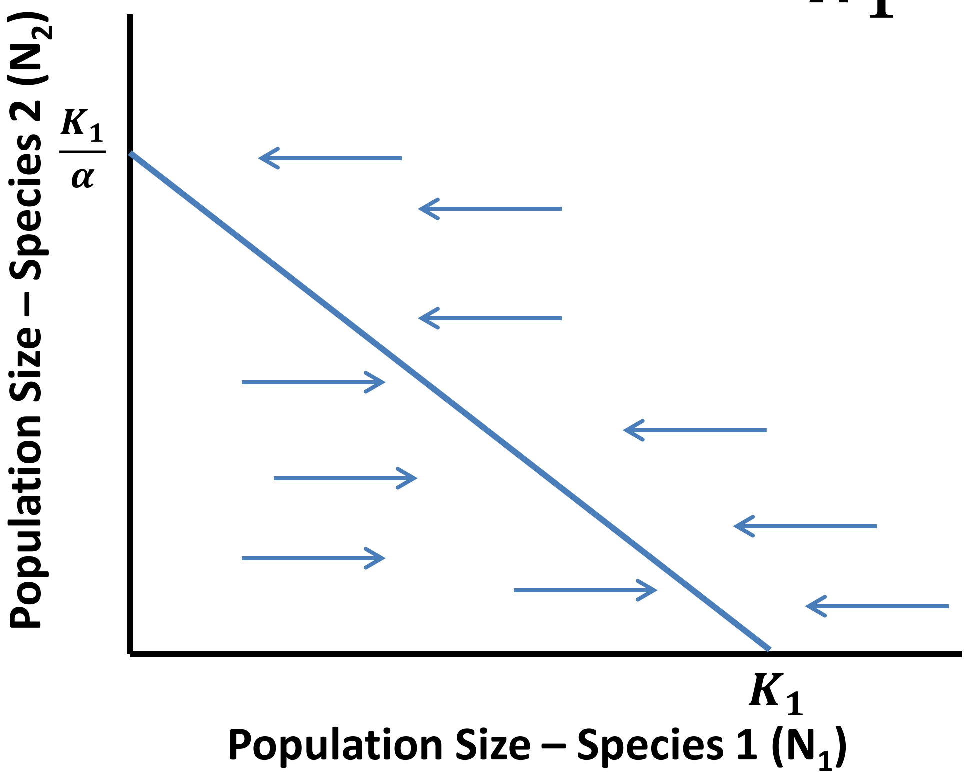


$$0 = K_2 - N_2 - \beta N_1$$

Isoclines and State-Space Graphs

- Stable equilibrium of population #1
- Use state-space graphs to represent $dN_x/dt = 0$ as a product of N_1 and N_2

$$\hat{N}_1 = K_1 - \alpha N_2$$



Growth of species #1

$$\hat{N}_1 + \alpha N_2 < K_1$$

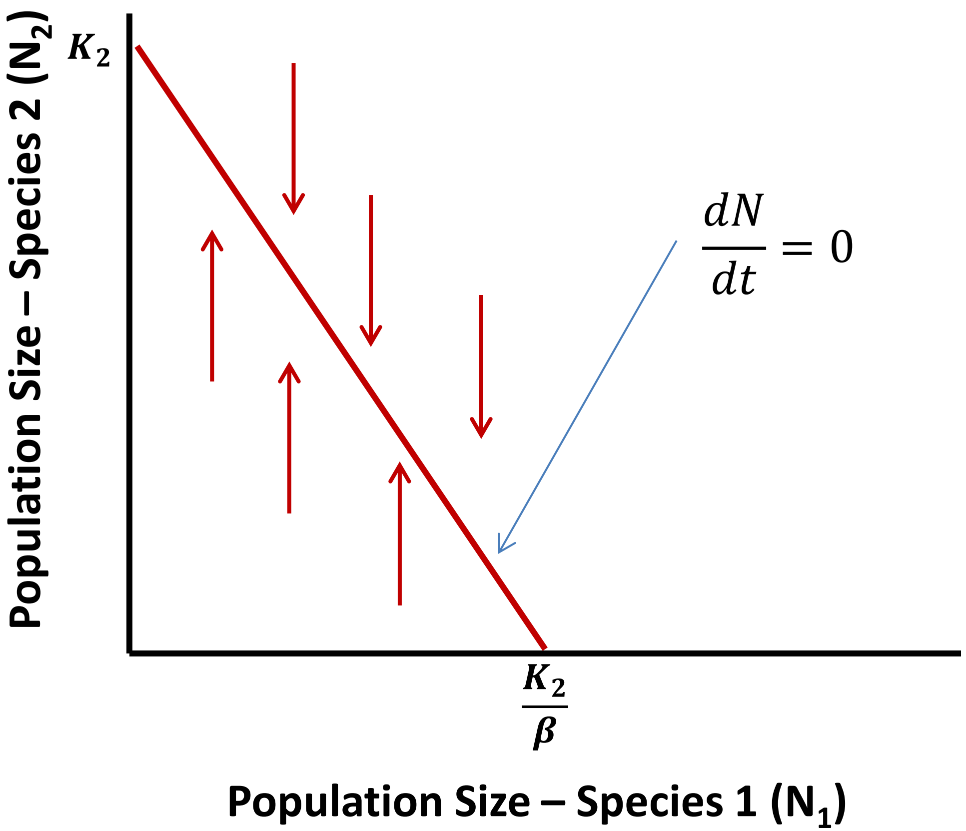
Decline of species #1

$$\hat{N}_1 + \alpha N_2 > K_1$$

Isoclines and State-Space Graphs

- Stable equilibrium population #2
  - Isocline for Species 2

$$\widehat{N}_2 = K_2 - \beta N_1$$



Growth of species #2

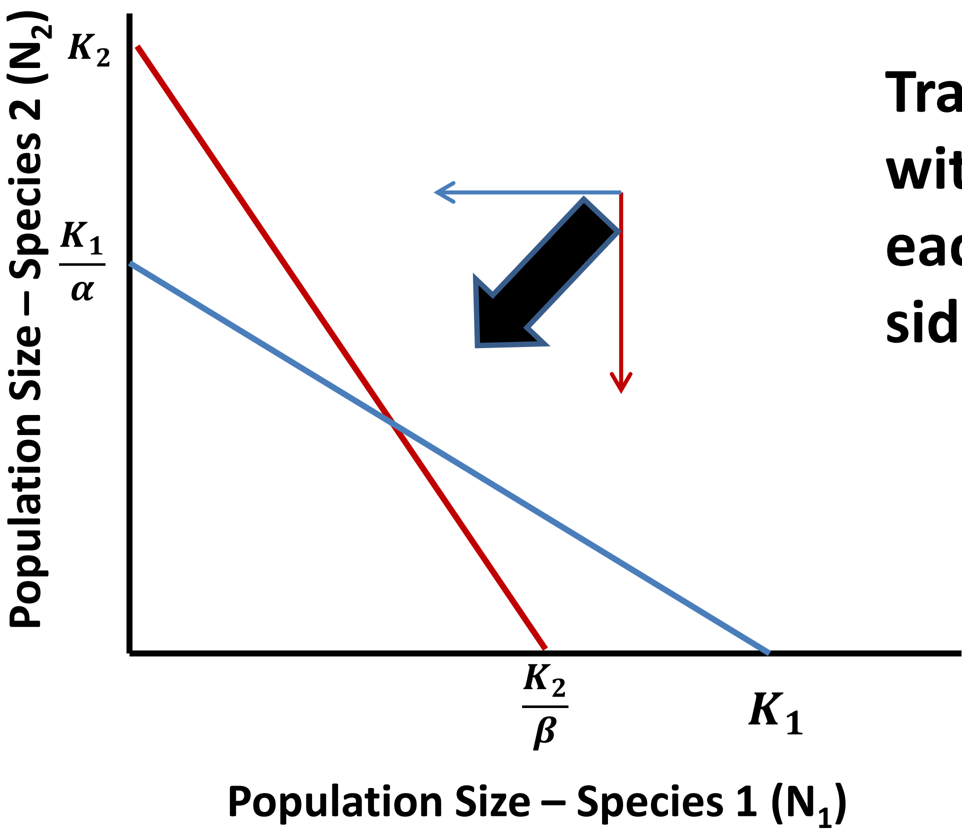
$$\widehat{N}_2 + \beta N_1 < K_2$$

Decline of species #2

$$\widehat{N}_2 + \beta N_1 > K_2$$

State-Space Competition Models

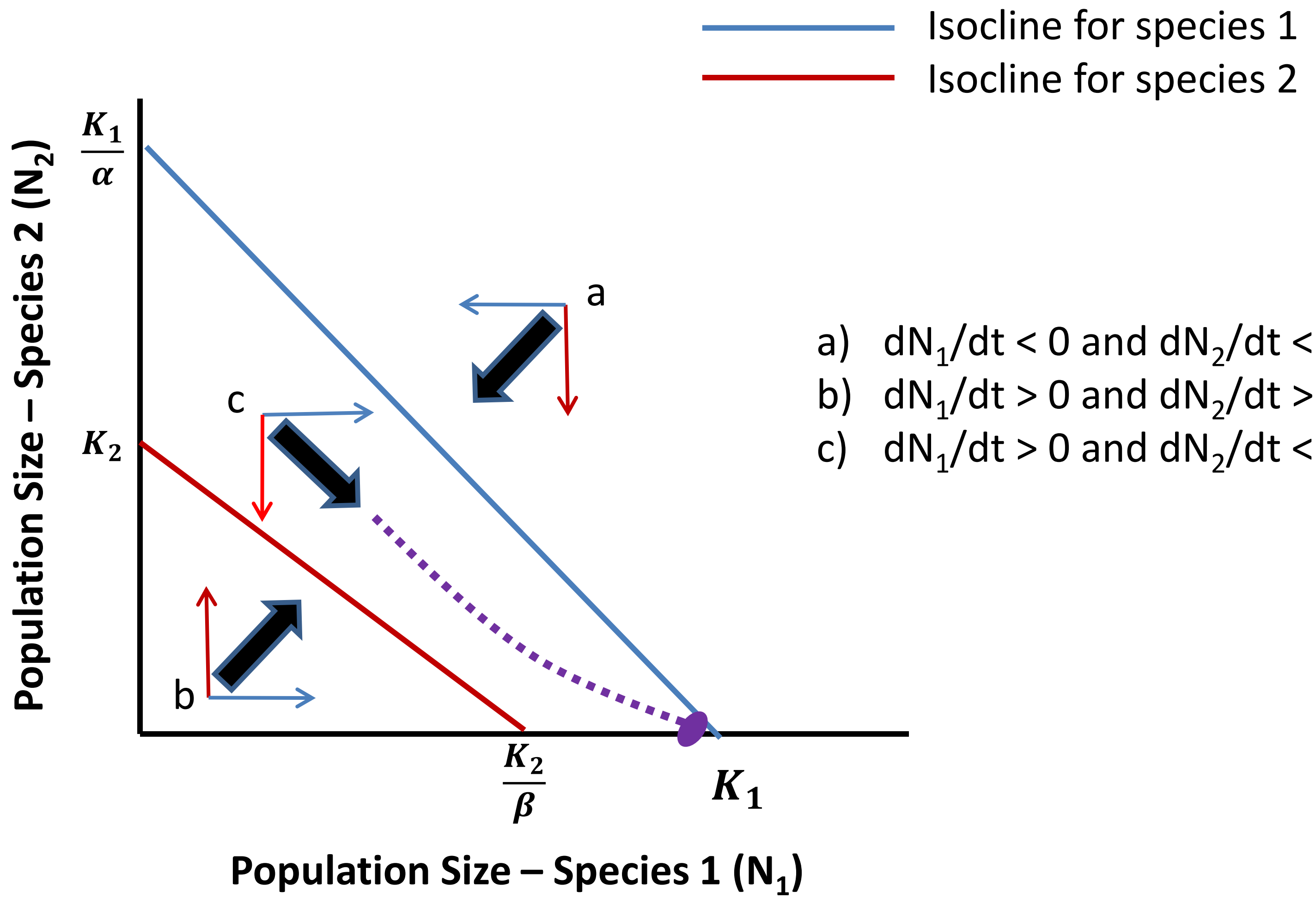
- Four Possible Outcomes of combining Isocline model lines representing



Track the vectors associated with the predicted change in each population on either side of their Isocline

Interspecific competition

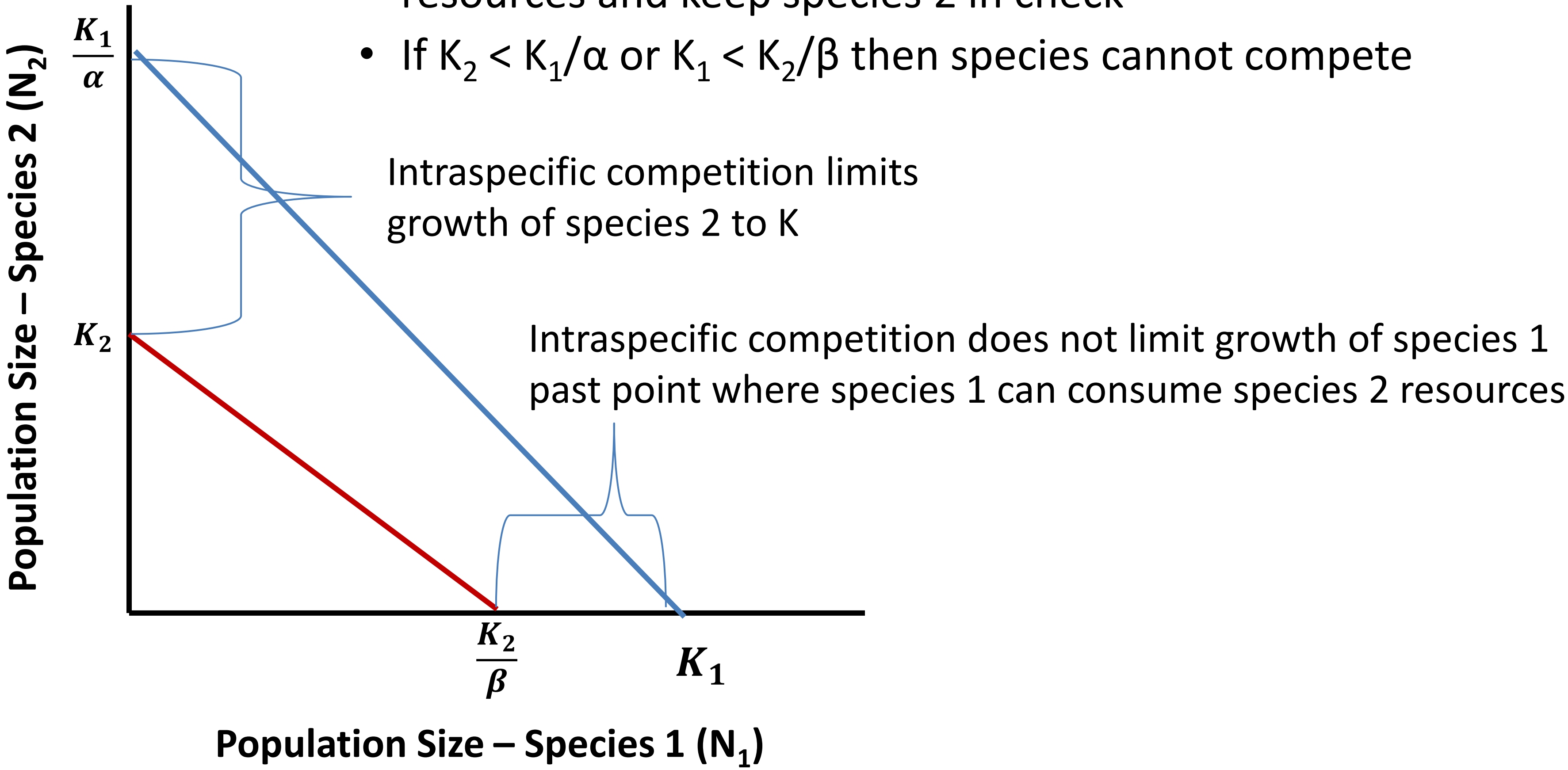
- Competitive exclusion of species 2 by species 1



a) $dN_1/dt < 0$ and $dN_2/dt < 0$
b) $dN_1/dt > 0$ and $dN_2/dt > 0$
c) $dN_1/dt > 0$ and $dN_2/dt < 0$

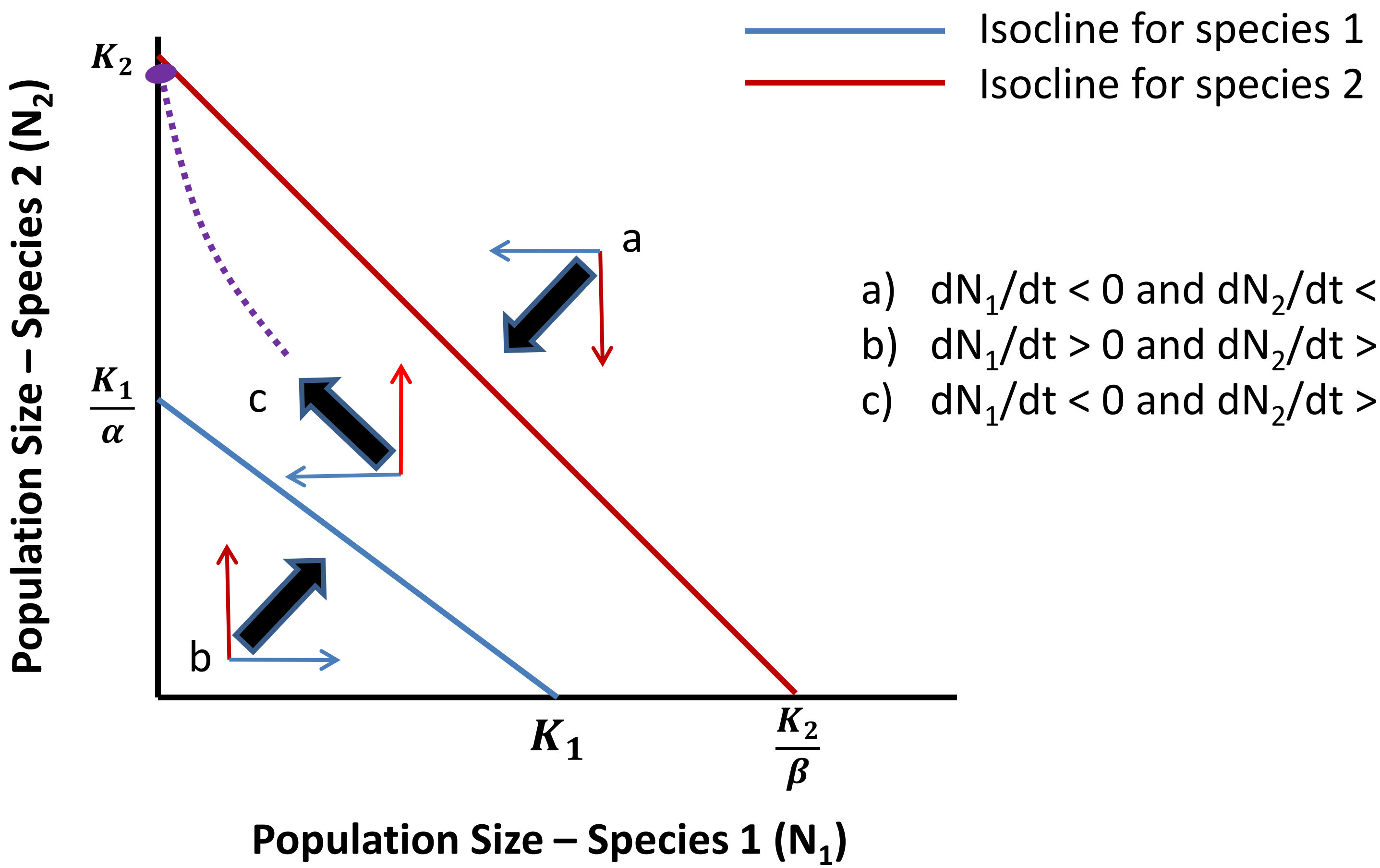
Interspecific competition

- Competitive exclusion of species 2 by species 1
  - K_1/α = number of species 2 required to consume species 1 resources and keep species 1 in check
  - K_2/β = number of species 1 required to consume species 2 resources and keep species 2 in check
  - If $K_2 < K_1/\alpha$ or $K_1 < K_2/\beta$ then species cannot compete



Interspecific competition

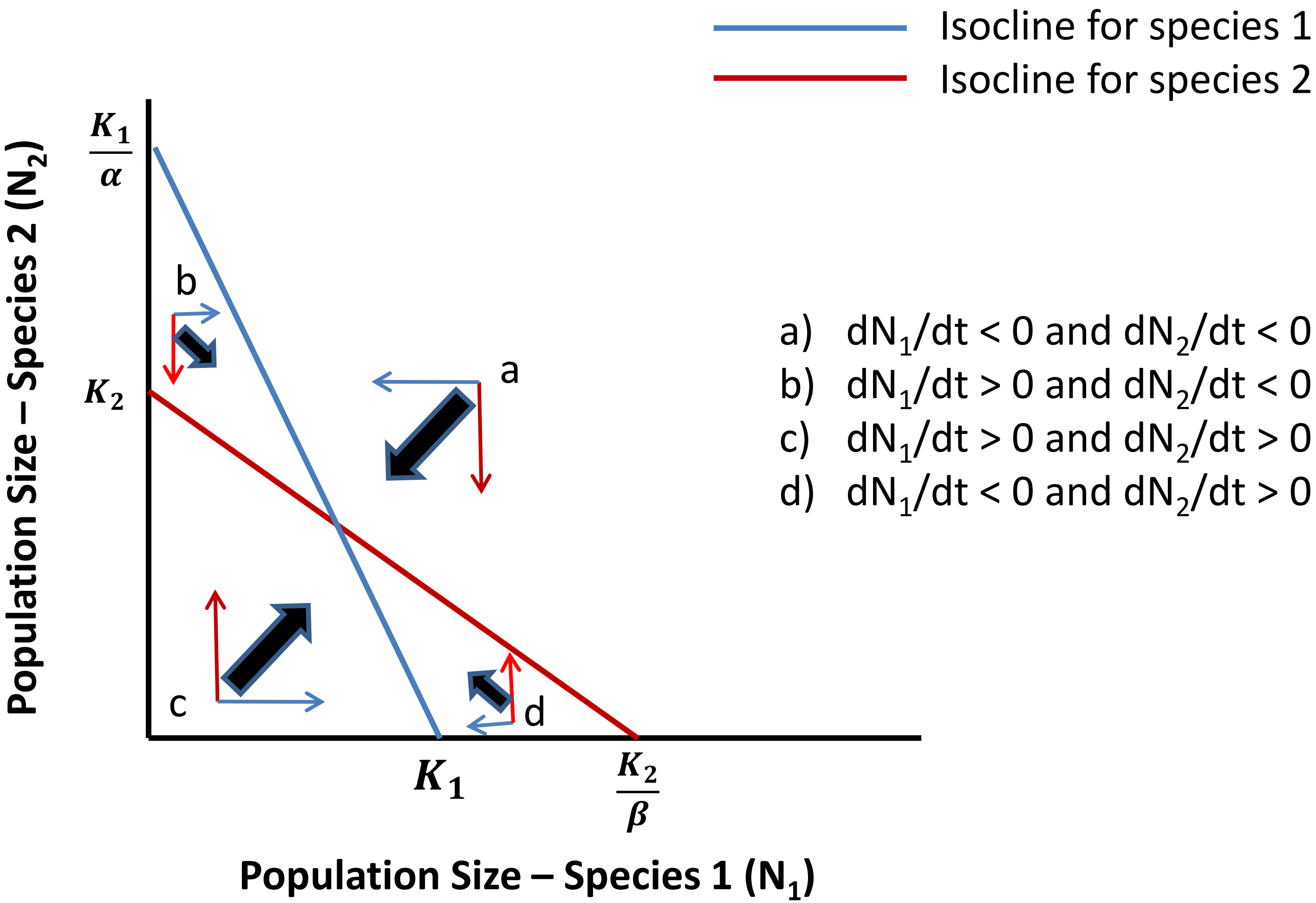
- Competitive exclusion of species 1 by species 2



a) $dN_1/dt < 0$ and $dN_2/dt < 0$
b) $dN_1/dt > 0$ and $dN_2/dt > 0$
c) $dN_1/dt < 0$ and $dN_2/dt > 0$

Interspecific competition

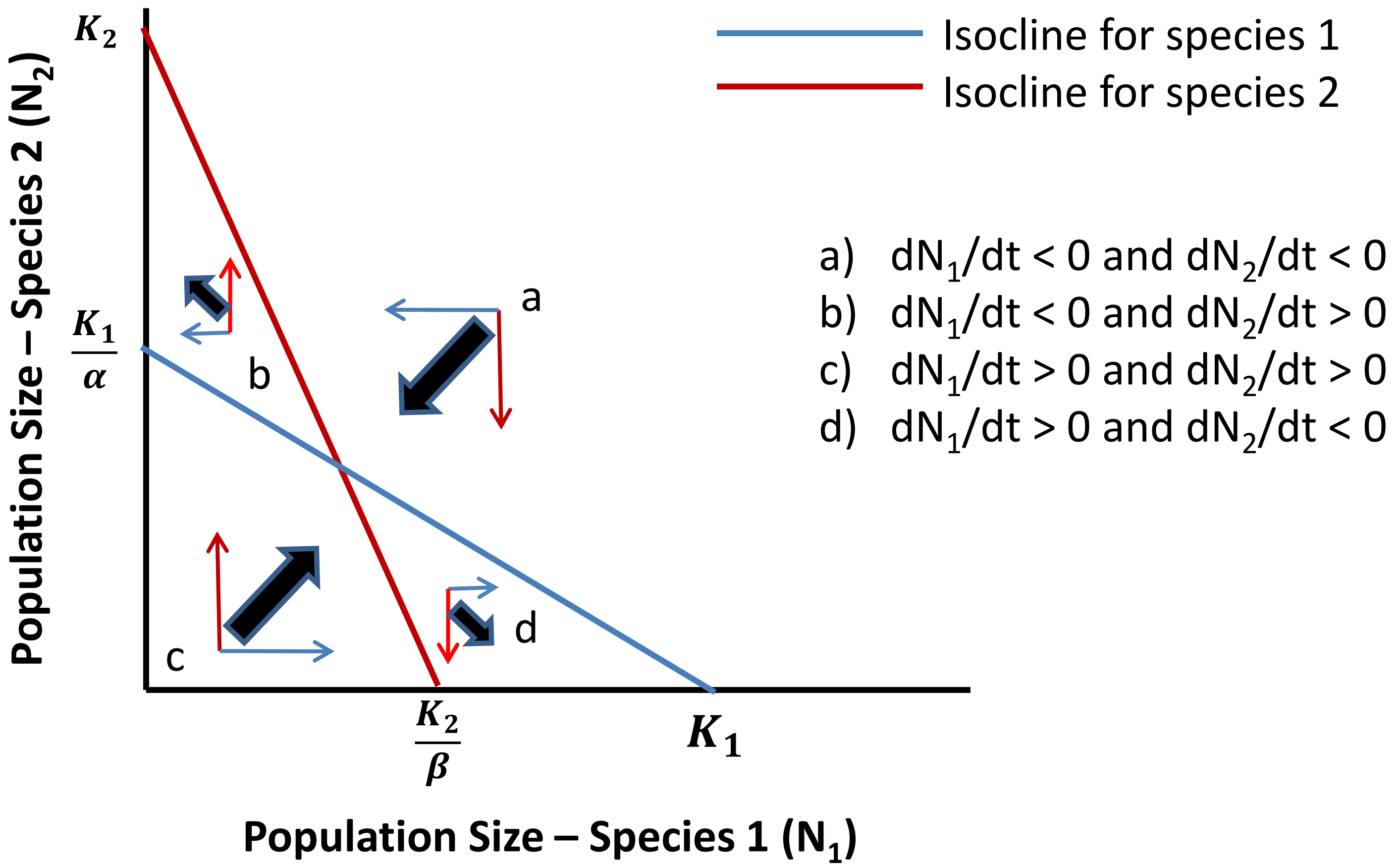
- Equilibrium between species 1 and species 2



a) $dN_1/dt < 0$ and $dN_2/dt < 0$
b) $dN_1/dt > 0$ and $dN_2/dt < 0$
c) $dN_1/dt > 0$ and $dN_2/dt > 0$
d) $dN_1/dt < 0$ and $dN_2/dt > 0$

Interspecific competition

- Unstable equilibrium for species 1 and 2
- Isoclines cross but $K_2 > K_1/\alpha$, and $K_1 > K_2/\beta$



Competitive Exclusion

- Scenario in which Species 1 could invade and establish in a population of Species 2.
- Assume:
  - Species 1's pop size is close to zero ($N_1 \approx 0$)
  - Species 2's pop size is close to carrying capacity ($N_2 \approx K_2$)
  - r_1 is positive
  - $\left(\frac{dN_1}{dt}\right)\left(\frac{1}{N_1}\right) = r_1\left(\frac{K_1 - 0 - \alpha K_2}{K_1}\right)$

Competitive Exclusion

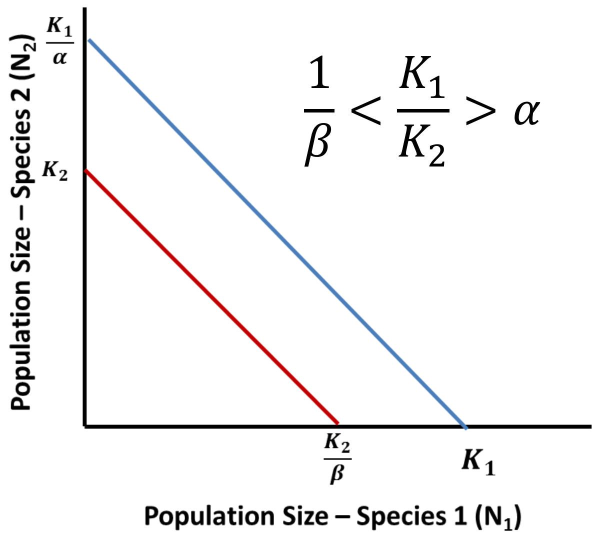
- Given
  - $\left(\frac{dN_1}{dt}\right)\left(\frac{1}{N_1}\right) = r_1\left(\frac{K_1 - 0 - \alpha K_2}{K_1}\right)$
  - N_1 can only increase when $\left(\frac{K_1 - \alpha K_2}{K_1}\right) > 0$ or:
  - $\left(\frac{K_1}{K_2}\right) > \alpha$
  - In other words, for Species 1 to invade and persist, the ratio of its carrying capacity to that of the competing species (K_2) must exceed the competitive effect of species 2 on species 1 (α).

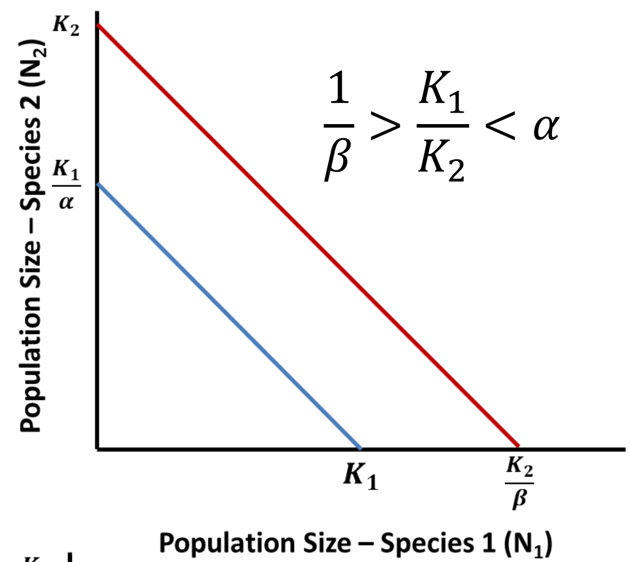
Competitive Exclusion and State-space models

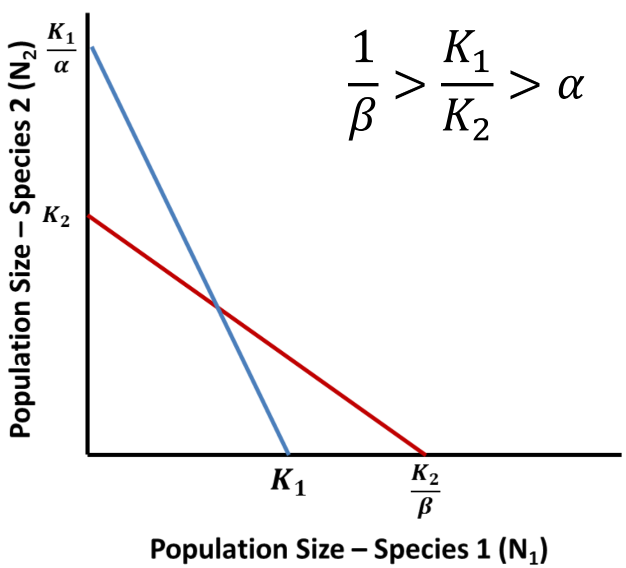
- Species 1 can invade when $\frac{K_1}{K_2} > \alpha$
  - but can't invade if $\frac{K_1}{K_2} < \alpha$
- Likewise, species 2 can invade when $\frac{1}{\beta} > \frac{K_1}{K_2}$
  - but can't invade if $\frac{1}{\beta} < \frac{K_1}{K_2}$

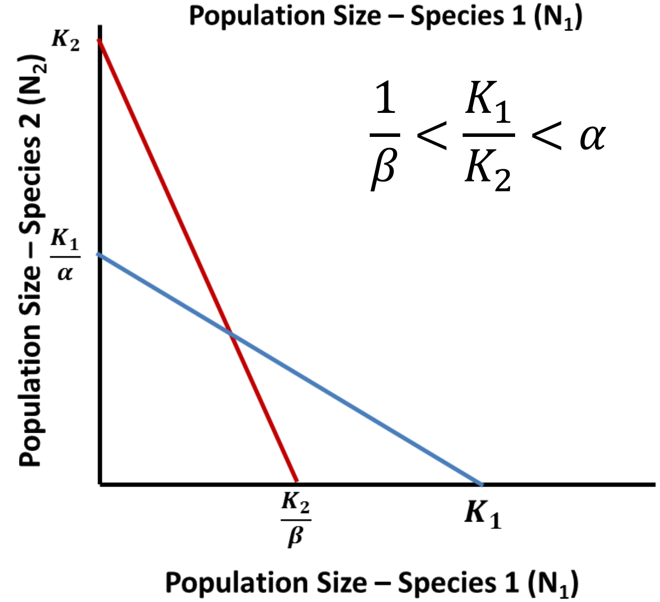
Four outcomes of competition

Stability? Invasion capability?









Principle of Competitive Exclusion

- Complete competitors can not Co-exist.
  - Two species with extremely similar ecology and physical attributes will have high competitive effects on each other (α and β are each close to 1)

    INTERSPECIFIC COMPETITION ≈ INTRASPECIFIC COMPETITION

  - As species diverge to utilize less overlapping resources (Resource Partitioning), their competitive effect on each other is lessened (α and β are both closer to zero).

    INTERSPECIFIC COMPETITION < INTRASPECIFIC COMPETITION

Model Assumptions

- Logistic model assumptions – no age or genetic structure, no migration, no time lags...

- Additional Assumptions:

1. Resources are in limited supply
2. Competitive coefficient (α/β) and carrying capacities (K_1 / K_2) are constants.
3. Density dependence is linear