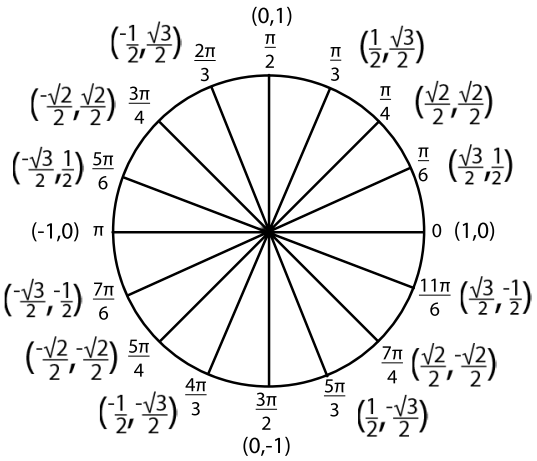
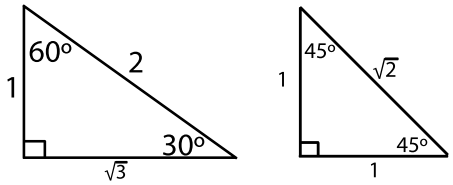
Trigonometry - Angles

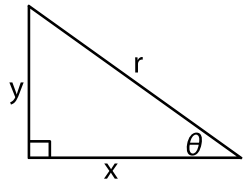
Unit Circle



Special Angle Triangles



Trigonometric Definitions



$$\sin\theta = \frac{y}{r}$$
$$\cos\theta = \frac{x}{r}$$
$$\tan\theta = \frac{y}{x}$$

Radian-Degree Conversion

To convert from degrees to radians we just need to muliply by $\pi/180$, and to convert from radians to degrees we multiply by $180/\pi$. More concisely:

$$1\text{Radian} = \frac{180}{\pi}\cdot\text{Degrees} \quad 1\text{Degree} = \frac{\pi}{180}\cdot\text{Radians}$$

In practice we use these ratios in one of the following forms:

$$1 = \frac{180}{\pi}\cdot\frac{\text{Degree}}{\text{Radian}} \quad 1 = \frac{\pi}{180}\cdot\frac{\text{Radian}}{\text{Degree}}$$

Example:

$$\begin{aligned} 15\text{ Deg} &= 15\text{ Deg}\cdot 1 \\ &= 15\text{ Deg}\cdot\left(\frac{\pi}{180}\cdot\frac{\text{Rad}}{\text{Deg}}\right) \\ &= \frac{15\cdot\pi}{180}\cdot\frac{\text{Deg}\cdot\text{Rad}}{\text{Deg}} \\ &= \frac{\pi}{12}\text{ Rad} \end{aligned}$$

Common Conversions

Degrees	0	30	45	60	90	120	135	150	180
Radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
Degrees	210	225	240	270	300	315	330	360	
Radians	$7\pi/6$	$5\pi/4$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$7\pi/4$	$11\pi/6$	2π	

Trigonometry - Identities

Fundamental Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \sin^2\theta + \cos^2\theta = 1$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \tan^2\theta - \sec^2\theta = -1$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta} \qquad \cot^2\theta - \csc^2\theta = -1$$

Identities for Negatives

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$
$$\cot(-\theta) = -\cot\theta$$
$$\sec(-\theta) = \sec\theta$$
$$\csc(-\theta) = -\csc\theta$$

Addition Formulae

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

Subtraction Formulae

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Cofunction Formulae

$$\cos(\pi/2-\theta) = \sin\theta$$
$$\sin(\pi/2-\theta) = \cos\theta$$
$$\csc(\pi/2-\theta) = \sec\theta$$
$$\sec(\pi/2-\theta) = \csc\theta$$
$$\cot(\pi/2-\theta) = \tan\theta$$
$$\tan(\pi/2-\theta) = \cot\theta$$

Double Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Half Angle Identities

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$

Squaring Identities

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Area of Triangle

$$K = \frac{1}{2}bc\sin A$$

$$K = \frac{1}{2}ac\sin B$$

$$K = \frac{1}{2}ab\sin C$$

$$K = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \dfrac{a+b+c}{2}$