Polynomial Division

Long Division

Find the quotient for: $\dfrac{x^4 - 7x^2 - x^3 + 6 + x}{x - 3}$

Notice the strong resemblance to the long division algorithm learned in elementary school.

- $x^4 - x^3 - 7x^2 + x + 6$ is the **dividend**.
- $x - 3$ is the **divisor** and the answer will be the **quotient**.

Process	Steps
Write the dividend and divisor in descending order.	$\dfrac{x^4 - x^3 - 7x^2 + x + 6}{x - 3}$
Set up the division in the typical form for long division (insert zero for missing terms in the descending order).	$x-3 \overline{)\, x^4 - x^3 - 7x^2 + x + 6}$
Determine what you have to multiply the leading term in the divisor by to get the leading term in the dividend. Place that result over the term in the dividend with the same exponent. (Ex. x^3 would be placed over the $-x^3$ in the dividend.)	$\begin{array}{r} x^3 \\ x-3 \overline{)\, x^4 - x^3 - 7x^2 + x + 6} \end{array}$
Multpily the top term by the divisor and place that as shown.	$\begin{array}{r} x^3 \\ x-3 \overline{)\, x^4 - x^3 - 7x^2 + x + 6} \\ \underline{x^4 - 3x^3} \end{array}$
Subtract and Drop! - be sure to remember to change the signs appropriately Drop down the next term.	$\begin{array}{r} x^3 \\ x-3 \overline{)\, x^4 - x^3 - 7x^2 + x + 6} \\ \underline{x^4 - 3x^3} \downarrow \\ 2x^3 - 7x^2 \end{array}$
Repeat the previous steps until there are no more terms to "Drop".	$\begin{array}{r} x^3 + 2x^2 - x - 2 \\ x-3 \overline{)\, x^4 - x^3 - 7x^2 + x + 6} \\ \underline{x^4 - 3x^3} \downarrow \\ 2x^3 - 7x^2 \\ \underline{2x^3 - 6x^2} \downarrow \\ -x^2 + x \\ \underline{-x^2 + 3x} \downarrow \\ -2x + 6 \\ \underline{-2x + 6} \\ 0 \end{array}$

Short Division (Synthetic Division)

While long division algorithm is general enough to handle any division needs, the short division algorithm is often much faster.

WARNING: We can *only* use short division when dividing by divisor of the form $(x + a)$. If the divisor is not of this form use long division.[1]

Find the quotient for: $\dfrac{2x^2 + x^3 - x - 2}{x + 2}$

Process	Steps
Write the dividend and divisor in descending order with zero for any missing term	$\dfrac{x^3 + 2x^2 - x - 2}{x + 2}$
Set the divisor equal to zero and solve.	$x + 2 = 0$, so $x = -2$
Write the number found in the previous step and the coefficients of the dividend in the format shown. Drop the first coefficient down as shown.	-2 | 1 2 -1 -2 1
Multply the number in the top left corner by the number in the bottom. Write it in the place shown (diagonally up to the right). Subtract and write the result in the bottom.	-2 | 1 2 -1 -2 -2 1 0
Repeat until you have written a number in the bottom right.	-2 | 1 2 -1 -2 -2 0 2 1 0 -1 | 0
The bottom row becomes the coeffificents for the quotient. The degree of the first term will be one less than the degree of the dividend. The number in the bottom right is the remainder. If there is a remainer it can be written over the divisor.	$x^2 + 0x - 1 + \dfrac{0}{x + 2} = x^2 - 1$
End with a final restatement of the problem with the quotient.	$\dfrac{x^3 + 2x^2 - x - 2}{x + 2} = x^2 - 1$

[1] If you are uncertain as to whether or not you may use short division, err on the side of caution and use long division.