Polynomial Division

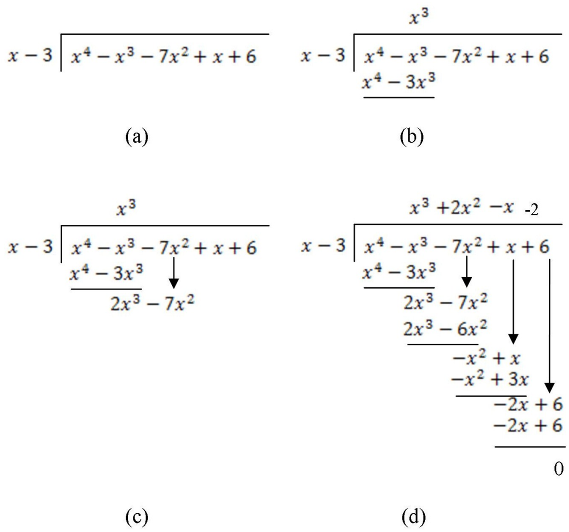
1 Long Division

Let's simplify the following expression:

$$\frac{x^4 - x^3 - 7x^2 + x + 6}{x - 3} \tag{1}$$

Let's look at an example. Notice the strong resemblance to the long division algorithm learned in elementary school.

Figure 1: Long Division



We will now be referring to Figure 1. The steps in long division for polynomials is very much long division for numbers.

1. We begin by setting up our page as in part (a).
2. Next determine how many times x goes into x^4 and write this at the top, multiply $x - 3$ by this number and write it below the dividend (b).
3. Subtract, carry down the next place (c) and repeat until you are left with the remainder (d).

NOTE: It is always best to write polynomials such as $x^3 + 1$ as $x^3 + 0x^2 + 0x + 1$ to keep things in alignment and prevent simple (but time consuming) errors.

2 Short Division

While long division algorithm is general enough to handle any division needs, the short division algorithm is often much faster.

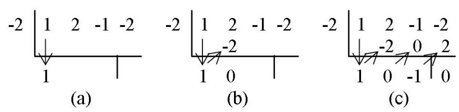
WARNING: We can *only* use short division when dividing by divisor of the form $(x + a)$. If the divisor is not of this form use long division.[1]

Let's work through an example:

$$\frac{x^3 + 2x^2 - x - 2}{x + 2} \tag{2}$$

Here's what a worked example looks like:

Figure 2: Short Division



We will now be referring to Figure 2. Here's a summary of the steps in short division:

1. First set up the page for short division as in Figure 2 (a). Notice that the coefficients are written at the top and we *negated* the 2. In general if we are dividing by $(x + a)$ is a factor, we write $-a$ at the top left corner. The negation is very important and missing it will produce false results. In case there are missing terms it is very important to write them in with their zero coefficients or again incorrect results will be produced.
2. Now we multiply the number in the top left corner by the number at the bottom, carry it diagonally and *add* it to the number above. Then write the results below. See Figure 2 (b).
3. Simply repeat the last step until the table is completely filled in (c).

The bottom numbers are the coefficients of the quotient and the bottom rightmost number (in the box) is the remainder.

Finally, we enter the coefficients and the remainder, then make our conclusion:

$$\frac{x^3 + 2x^2 - x - 2}{x + 2} = x^2 + 0x - 1 + \frac{0}{x + 2} = x^2 - 1 \tag{3}$$

[1] If you are uncertain as to whether or not you may use short division, err on the side of caution and use long division.