Academic Success Centre
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Derivatives

Definition of derivative:

$f'(x) = \lim_{x \to a} \frac{(f(x)-f(a))}{x-a}$ or $f'(x) = \lim_{h \to 0} \frac{(f(x+h)-f(x))}{h}$

Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

Constant Rules: $\frac{d}{dx}\left(c * f(x)\right) = c\frac{d}{dx}\left(f(x)\right)$

Product Rule: $\frac{d}{dx}\left(f(x) * g(x)\right) = \frac{d}{dx}\left(f(x)\right) * g(x) + f(x) * \frac{d}{dx}(g(x))$

Reciprocal Rule: $\frac{d}{dx}\left(\frac{1}{g(x)}\right) = -\frac{\frac{d}{dx}g(x)}{g^2(x)}$

Quotient Rule: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{\left(\frac{d}{dx}f(x)\right)*g(x)-\left(\frac{d}{dx}g(x)\right)*f(x)}{g^2(x)}$

Chain Rule: $\frac{d}{dx}(f\left(g(x)\right) = \frac{df}{dx}(g(x)) * \frac{dg}{dx}(x)$

Trig Derivatives

$\frac{d}{dx}(\cos(x)) = -\sin(x)$

$\frac{d}{dx}(\sin(x)) = \cos(x)$

$\frac{d}{dx}(\tan(x)) = \sec^2(x))$

$\frac{d}{dx}(\sec(x)) = \tan(x)\sec(x)$

$\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$

$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{(1-x^2)}}$

$\frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$

Other Derivatives

$\frac{d}{dx}(a^x) = a^x * \ln(a)$

$\frac{d}{dx}(e^x) = \mathrm{e}^{\mathrm{x}}$

$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, x > 0$

$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, x \neq 0$

$\frac{d}{dx}(\log_a x) = \frac{1}{x * \ln(a)}, x > 0$

Curve Sketching Steps

Use these steps when curve sketching. Not all steps are always necessary. For example, some functions have no symmetry.

1. **Domain**
   Determine the set of values of x where f(x) is defined. Common examples include:
   $\sqrt{x}, x \geq 0$ $\ln x\,, x > 0$ $\tan x\,, x \neq \frac{n\pi}{2}, n \text{ is an integer}$

2. **Intercepts**
   Find the intercepts of the function. The y-intercept of the function f(x) is f(0). The x-intercepts can be found by setting f(x)=0 and solving for x.

3. **Symmetry**
   If f(x)=f(-x) for all x in the domain, then f is an even function.
   If f(x) = -f(-x) for all x in the domain, then f is an odd function.
   If f(x+p) = f(x) for all x in the domain, and p is a positive constant, then f is a periodic function.
   Examples: cos(x) is even, sin(x) is odd, and sin(x) = sin(x+2π), so sin(x) is 2π periodic.

4. **Asymptotes**
   A line y=L is a horizontal asymptote if $\lim_{x\to\infty} L$ or if $\lim_{x\to-\infty} L$. L must be not be $\pm\infty$.
   The line x=a is a vertical asymptote if one of these statements is true:
   $$\lim_{x\to a^+} f(x) = \pm\infty \qquad \lim_{x\to a^-} f(x) = \pm\infty$$
   Check your text for information about slant asymptotes.

5. **Intervals of Increase or Decrease**
   Find f'(x) and find the intervals on which f'(x) is positive and negative. Set f'(x) = 0 and solve for x. Also find the points where f'(x) is not defined, but f(x) still is. Subdivide the real numbers into different intervals divided by these points of x. Then use test numbers or use a polarity chart to determine whether f'(x) is positive or negative in each interval.

6. **Local Maximum and Minimum Values**
   Find the critical numbers of f (the points where f'(x)=0) and use the First Derivative Test. If f'(x) changes from positive to negative at a point c, then c is a local maximum. If f'(x) changes from negative to positive at a point c, then c is a local minimum.

7. **Concavity and Points of Inflection**
   Compute f''(x) and determine the intervals on which f''(x) is positive (concave up) and negative (concave down). If the concavity changes at a point, then it is an inflection point. If there is no concavity change, it is not an inflection point.

8. **Sketch the Curve**
   Use the information obtained above to sketch the curve. Asymptotes can be drawn in as dashed lines. Put in the points for intercepts, maximums, minimums and inflection points; then try your best to draw a line through these points with the appropriate concavity.