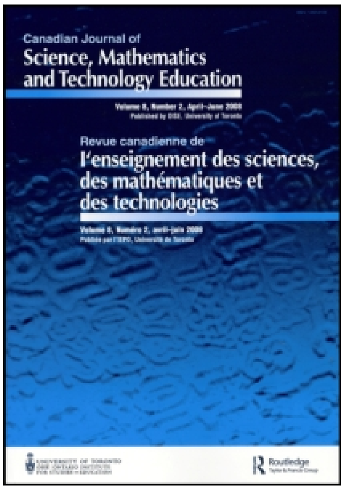

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Assessment on Previous Course Work in Calculus and Subsequent Achievement in Calculus at the Post-Secondary Level

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Abstract: There is limited research examining the effect of antecedent work in calculus and subsequent achievement in post-secondary calculus. This study examines the effects of prior coursework in calculus, often a locally developed secondary school calculus class, and ensuing achievement in a first-year university calculus class. Analysis of covariance confirms that students who had successfully completed antecedent course work in calculus are achieving higher final grades than their equally capable peers, even after gender differences and prior achievement are accounted for in the ANCOVA model. Sequential linear regression results substantiate the significance of antecedent coursework as a predictor of final grades.

Résumé: Il y a encore peu d'études de recherche analysant les effets du travail scolaire en calcul sur les résultats obtenus dans cette matière dans les cours de niveau postsecondaire subséquents. La présente étude se penche sur les effets du travail effectué précédemment dans un cours de calcul, en général un cours de calcul mis sur pied dans une école secondaire, et les résultats qui ont suivi dans un cours de calcul de première année universitaire. Une analyse des covariances confirme que les étudiants de niveau postsecondaire ayant suivi avec succès des cours précédents en calcul obtiennent de meilleurs résultats de fin d'année dans cette matière que leurs pairs de même niveau, même si on tient compte des différences de sexe et de résultats antécédents grâce au modèle ANCOVA. Les résultats de la régression linéaire séquentielle fournissent des preuves de l'importance du travail scolaire antécédent comme indice des résultats universitaires de fin d'année.

INTRODUCTION

A first course in calculus, usually differential calculus, is traditionally one of the most challenging courses encountered by students at the tertiary level. Typically, a good predictor of achievement in an advanced mathematics course is the student's final grade in the prerequisite course (Ma, 2000). This premise was supported by the authors' observations. Students who earned A grades, over 85% (British Columbia Ministry of Education, 2006), in Mathematics 12 generally earned high B or A grades in their first course in calculus (Calculus I). Mathematics 12, taught at the secondary

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TABLE 1
BC Secondary School Grading System

Letter Grade	*Percentage*
A	86–100
B	73–85
C+	67–72
C	60–66
C−	50–59
F	0–49

school level, is the prerequisite course to Calculus I. However, over the past decade, the pattern of grades has changed in the Calculus I course offered at this British Columbia institution. There appeared to be more variability in the grades with a number of students excelling in the course, a number performing poorly, and only a small percentage earning the typical B grades normally associated with the course. Tables 1 and 2 contain a breakdown of the percentages associated with letter grades used in this study at the secondary school level and at the institutional level. In efforts to ascertain the underlying cause for the observed trends, one instructor informally asked a class about their mathematics backgrounds. She noted that many of the A-grade students reported that they had already taken a calculus class either through secondary school, college, or university. This unexpected response led the instructor to investigate this effect more thoroughly because this has implications for how calculus is taught and how students achieve in a first course in calculus in university.

Background

A number of factors may contribute to the observed changes in calculus grades. For example, the introduction of graphing calculators in secondary school has raised concerns both in secondary

TABLE 2
UNBC Grading System

Letter Grade	*Percentage*
A+	90–100
A	85–89.9
A−	80–84.9
B+	77–77.9
B	73–76.9
B−	70–72.9
C+	67–69.9
C	63–66.9
C−	60–62.9
D+	57–59.9
D	53–56.9
D−	50–52.9
F	0–49.9

and post-secondary institutions with some arguing that student skills are weakened as a result of calculator usage, a topic still debated among mathematics educators. Another reason may be that more students are required to take mathematics as part of degree requirements, leading to greater diversity in student class composition. A third contributing factor may be the several revisions to the secondary school mathematics curriculum that have evolved across Western Canada over the past two decades (Ahn, 2004; British Columbia Ministry of Education, 2009). The need for improved mathematical skills in students at the secondary school level has driven the transformations in curriculum; conversely, these changes have contributed to more varied backgrounds skills noted among students entering post-secondary institutions. Some students enter post-secondary school with excellent algebraic skills, whereas others may be lacking certain skills as some concepts have been missed or omitted from secondary school mathematics classes. This results in a mismatch between post-secondary educators' expectations and the actual level of preparedness of students.

One particularly significant change in secondary school mathematics revisions was the removal of an introductory chapter on calculus from the Mathematics 12 curriculum in 1999 (Ahn, 2004). Prior to the removal of the calculus chapter from Mathematics 12, most students enrolling in Calculus I had similar backgrounds—Mathematics 12 with a C− grade or better (the C− requirement may vary across institutions and may also depend on year of entrance). Subsequently, a student's Mathematics 12 had been a good indicator of outcome in Calculus I, with students having similar backgrounds that included an introductory chapter in calculus. The removal of calculus from Mathematics 12 had another effect in secondary schools. In efforts to prepare students for post-secondary education, increasing numbers of secondary schools began offering locally developed calculus courses and/or advance placement calculus courses. Many students complete a locally developed Calculus 12 course, thus providing them with an extensive introduction to calculus in preparation for university calculus. Other students may complete an advance placement calculus course, allowing them to earn credit for the equivalent of Calculus I prior to enrolment in college or university. However, students completing an advance placement calculus course are not required to report their grades even if a maximum score is achieved. Some students elect not to take the advance placement credit and instead opt to take Calculus I, recognizing the probability of earning an easy A grade. As increasingly more school districts offered secondary school calculus courses and more students took advantage of these classes, the diversity of student backgrounds entering a first course in calculus intensified. Unfortunately, students without antecedent course work in calculus appeared to be at a disadvantage. Note, we cannot assume that more capable students took calculus in secondary school. Nor may we assume the reverse—that less capable students took calculus in efforts to improve their chances in first-year calculus at the post-secondary level. Also, we do not know of gender bias in terms of selection of antecedent calculus course work. Nonetheless, the results of the informal survey supported the notion that antecedent coursework in calculus was the most significant indicator of final grades in Calculus I and formed the basis for the question to be analyzed in this article.

Objectives

The objective of this study was to determine the effect of antecedent coursework in calculus, often a locally developed calculus class offered in secondary school, and subsequent achievement in a first-year calculus class at the university level after accounting for gender and ability. The research question is, "Is antecedent course work in calculus a significant predictor of final grades in a first course in calculus at the post-secondary level?"

The effect of previous calculus course work on a final grade in a first course in calculus may seem obvious or commonsensical. However, previous research examining this topic was limited; much of the research was outdated and, furthermore, the results were often contradictory. Some of the research suggested that an antecedent calculus course did not improve outcomes in a first-year calculus class (Rosenstein, 2005; Wilhite, Windham, & Munday, 1998). However, other research (Jones, 1987; Ma, 2000) claimed otherwise. Ma's research indicated that every advanced mathematics course (courses in senior grades) contributed significantly to achievement in following courses. The University of British Columbia (UBC) stated in a 2005 report on schools, "Students with a high school calculus course continue to do much better than students who had not taken a high school calculus course. In fact, the gap between these groups of students has widened significantly." However, they do not indicate how this conclusion was determined or how the data were obtained (University of British Columbia, 2007, Comments section). Given the inconclusiveness of previous research, the changing student demographic at universities across Canada, the need for mathematical competency, and the lack of current evidence, further empirical research into this question was warranted.

METHOD

Data Source

The course selected for analysis was Calculus I (MATH 100) offered in the fall semester of 2004. Data used in the analyses included the final grades, as percentages, earned by students enrolled in the course; the students' previous Mathematics 12 (or equivalent) course grade; previous calculus course grades; and gender. These were obtained from the registrar's office with ethics approval. The antecedent calculus category included locally developed secondary school calculus courses, advancement placement calculus courses, and college transfer courses of insufficient content for credit as Calculus I as determined by the institution.

Participants

Total enrollment for Calculus I was 167 students. Twenty-seven of these were not included in the analyses because they either withdrew (9 students) or the researchers were unable to determine whether they had previously taken a calculus course (18 students), leaving a sample that consisted of 140 students. There were 64 students in the antecedent calculus group. Of these, three students had previously passed Calculus I but enrolled again in an attempt to improve their grades in the course. One transfer student had completed a calculus course with insufficient content for transfer credit. Another 8 students had completed our institutional course, Calculus for Non-Majors, subsequently changed programs, and were required to take Calculus I. These students were also included in the antecedent calculus category.

Gender, always of interest when examining data in mathematics, was not notably one-sided: 63 female, 77 male. The mean final course grade in Calculus I for males was 64%; for females, 73%. This may be unexpected because there has long been a perception that males achieve higher grades in mathematics, although more current research (Gallagher & Kaufman, 2005; Linn & Kessel, 1996; Randhawa, 1994) suggests otherwise. As such, gender is included in the analysis to examine any effects that gender may add.

RESULTS AND DISCUSSION

An examination of the mean final grades and a two-tailed *t*-test for equality of means formed the basis for a preliminary assessment to compare students who have antecedent calculus course work and students who have not or who may have failed or withdrawn from a calculus course. The mean final grade of the antecedent calculus group was 76.6% with SD = 16.9. The mean final course grade of students who did not have antecedent calculus course was 60.6% with SD = 21.5.The difference in mean final course was significant ($t = 4.9$, df = 138, $p < .001$) with the students who had antecedent calculus course outperforming their counterparts by 16%. In terms of letter grades this is the difference between earning a C− to a B/B+ (based on our institution's grading system). As notable as these results are, we recognize that these differences do not account for the effects of gender, nor do they take into consideration a student's previous mathematical achievement. Both of these factors, as yet unaccounted for, are likely to result in selection bias effects.

In efforts to control for these considerations, student achievement in Mathematics 12 was used as a covariate in the following analyses. The use of the Mathematics 12 covariate resulted in the adjustment of the final grades to what they would be if everyone had the same incoming Mathematics 12 grade, thus creating a method to examine the effects of antecedent course work in calculus. The Mathematics 12 grade was selected in view of the fact that previous research has established mathematics scores to be a good indicator of achievement in subsequent mathematics courses (Jones, 1987; Ma, 2000). The Mathematics 12 variable included grades from Principles of Mathematics 12 (British Columbia), Advanced Functions & Introduction to Calculus course, Mathematics 30 (Alberta), and MATH 115 (university Pre-Calculus). All of these courses met equivalency requirements for Mathematics 12 at this institution. British Columbia mathematics secondary school grades were provided as letter grades ranging from a C− to an A with all possible options as follows: A, B, C+, C, C−. Note that a C− grade was needed to meet the minimum entrance requirement for Calculus I. Institutional grading for MATH 115 also included B+, B− and A+, A− grades. These were grouped as B and A grades, respectively. Other equivalent courses were also grouped according to a corresponding Mathematics 12 grade. There were 2 students enrolled in Calculus I with no information available for this variable, thus reducing the sample size to 138. Gender was analyzed to explore its importance in the analysis.

To answer the research question, "Was antecedent coursework in calculus a useful predictor of final grades?" an analysis of covariance (ANCOVA) was performed to examine the data for significant differences in final grades between the two groups, those with antecedent calculus and those without, using adjusted means. Table 3 summarizes the adjusted mean final grades obtained through ANCOVA, broken down by gender and calculus background.

There were no significant interaction effects occurring in the ANCOVA ($p = .778$). The Mathematics 12 covariate was significant ($p < .0005$), validating its use. Both antecedent calculus and gender displayed statistically significant differences ($p = .001$ and $p = .014$, respectively). A

TABLE 3
Mean Final Grades (Gender and Antecedent Calculus)

Gender	*Antecedent Calculus*	*M*	*SE*	*N*
Male	No	60.2	2.7	43
	Yes	69.7	3.0	33
Female	No	66.8	3.1	31
	Yes	77.9	3.1	31
Overall	No	63.5	2.1	74
	Yes	73.8	2.2	64

Cohen's *d* measure was calculated to determine the practical significance of both factors. A value of 0.49 was obtained for antecedent calculus, corresponding to a 10.3 percentage point difference in mean final grades favoring students with antecedent calculus. This is the difference between a B+ grade and an A+ grade, a considerable variance for any student. A Cohen's *d* value of 0.35 was obtained for gender, comparable to a 7.4 percentage point difference in mean final grade favoring females.

The use of ANCOVA provided a simultaneous analysis of all three variables, antecedent calculus, Mathematics 12 achievement, and gender. The results also ruled out any gender/antecedent calculus interaction effects. However, we still wished to determine the value of antecedent course work in calculus as a predictor of final grades in a course such as Calculus I, after removing the effects of ability and gender. We carried out a sequential linear regression analysis, comparing two models, to determine the value of antecedent course work in calculus for prediction of final grades in Calculus I. The first model contained the Mathematics 12 grade and gender. Results of the regression confirmed that the model was significant for prediction of final grades ($p < .0005$). The Mathematics 12 grade was significant in the model ($p < .0005$, Table 4) and the strongest predictor ($\beta = .434$, Table 4). This was expected and once more confirmed our use of the Mathematics 12 grade as a covariate. Gender was also significant in the model ($p < .05$).

TABLE 4
Regression Models

Model	*B*	*SE*	*β*	*t*	*p*
Model 1					
Constant	22.7	6.2		3.7	.000
Gender	7.5	3.0	.179	2.5	.014
Mathematics 12	8.7	1.2	.503	7.0	.000
Model 2					
Constant	23.7	6.0		3.9	.000
Gender	7.3	2.9	.173	2.5	.014
Mathematics 12	7.5	1.2	.434	6.0	.000
Antecedent calculus	10.3	3.1	.244	3.4	.001

TABLE 5
Regression Model Differences Summary

Model	*df*	*R*	*Adj* R^2	*SE*	*F*	*P*
Model 2: gender, Mathematics 12, antecedent calculus	3	.597	.342	17.1	24.7	.000
Model 1: gender, Mathematics 12	2	.548	.290	17.8	29.1	.000
Difference	1		.052			

The second model, also significant ($p < .0005$), included the antecedent calculus variable. In model 2, we observed that antecedent calculus was a significant predictor of final grades in Calculus I ($p = .001$).

Further regression statistics are presented in Table 5. This table contains a summary of results for both models; the differences between the model results are used to assess the percentage of variation in final grades explained by the addition of the antecedent calculus variable.

The amount of variation (adjusted R^2) explained by model 2 is 34.2%, an improvement of 5.2% over model 1. To determine whether the change in model was significant, we tested the null hypothesis of no increase in R^2 using $F_{inc} = \frac{(R^2_{wi} - R^2_{wo})/m}{(1-R^2)df_{res}}$ (Tabachnik & Fidell, 2007) where F_{inc} is the incremental F ratio; R^2_{wi} is the multiple R^2 achieved with the added variable; R^2_{wo} is the multiple R^2 achieved without the additional variable; m is the number of added independent variables in the new model ($m = 1$ for the antecedent calculus variable added to model 2); and $df_{res} = N - k - 1$. We obtained a value for $F_{inc} = 6.99$ (df = 1, 134), a significant change in F_{inc} ($p < .01$). Adding the antecedent calculus variable significantly improved the model for predicting final grades even after accounting for gender differences and prior achievement in Mathematics 12.

We also wished to quantify the contribution of each predictor, Mathematics 12, gender, and antecedent calculus, to final grades in Calculus I. Using the results of the linear regression, we obtained the following equation for prediction:

$$Y = 23.7 + 7.3X_G + 7.5X_{M12} + 10.3X_{AC}$$

where X_G represents gender, X_{M12} represents the Mathematics 12 grade, and X_{AC} represents the antecedent calculus variable. X_G is given a value of 0 for males and a value of 1 for females. X_{M12} is given values for letter grades as follows: A = 5, B = 4, C+ = 3, C = 2, and C− = 1 (a C− grade is the minimum requirement for admission to Calculus I at this institution). The coefficient of X_{AC} is of most interest. X_{AC} is given a value of 0 if the student does not have antecedent calculus course work and a value of 1 if the student has prior course work in calculus. The value 10.3 represents an increase of 10.3 percentage points in final grade in a first course in calculus at university given that the student has successfully completed antecedent course work in calculus ($X_{AC} = 1$). In comparison, there is an increase of 7.5 percentage points in final grade for each letter grade improvement in a student's Mathematics 12 mark and an increase of 7.3 percentage points in final grade favoring females. The following examples demonstrate use of the prediction equation. A female student with a B grade (73–85%) in Mathematics 12 and antecedent calculus has a predicted score of $Y = 23.7 + 7.3(1) + 7.5(4) + 10.3(1)$ or 71.3%. Without antecedent calculus course work, this same female student with a B grade in Mathematics 12 is predicted to earn a final grade of $Y = 23.7 + 7.3(1) + 7.5(4) + 10.3(0)$ or 61%. This is the difference between a B− and a C− at this institution. Another example demonstrates the equation for a male student. If a male student earned a B in Mathematics 12 and he completed a locally developed calculus course, his predicted score in Calculus I is $Y = 23.7 + 7.3(0) + 7.5(4) + 10.3(1) = 64.0\%$ or a C grade. Without the antecedent calculus course, his predicted score is $Y = 23.7 + 7.3(0) + 7.5(4) + 10.3(0) = 53.7\%$ or a D grade. However, it is important to recognize these are predicted scores only. Both males and females will have actual scores that would be expected to vary around these predicted scores.

CONCLUSIONS

Students with antecedent course work in calculus, taking Calculus I, earned higher grades than students without prior calculus coursework, even after controlling for mathematical ability and gender. Thus, the observed differences between students who have antecedent calculus and those who do not cannot be dismissed as due to more capable students taking the course or due to a gender bias. From a practical viewpoint, antecedent course work in calculus is seen to provide students a significant advantage in Calculus I.

Implications for Practice

Because many first courses in calculus were originally designed and are still intended for students with the equivalent of Mathematics 12, changes should be considered to ensure an equitable environment for those with and without antecedent calculus. Post-secondary institutions can create courses that take into consideration the background of the student. For example, one section may offer 4 hours of contact time weekly, versus 3 hours, for students new to calculus. This would allow students without a calculus background to take a course that provided more instructional contact time while covering the same material. Some institutions have already initiated courses aimed at addressing this difference in background. The University of British Columbia (2007) offers six different first calculus courses, two of which are 4-credit hour and four of which are 3-credit hours. The 4-credit hour courses are only open to students who have not passed a previous calculus course. The students in the 4-credit hour courses take common final exams with students in the parallel 3-credit hour courses. Simon Fraser University offers three distinct first-year calculus courses, each of which is 3 credit hours (Simon Fraser University, 2008). One is for biological sciences, one is a regular calculus class, and the third is similar to the second but has 4 lecture hours per week instead of 3. Clearly, our perception that students with antecedent coursework in calculus lent itself to improved final grades was not isolated to our institution.

There are other options to consider. For example, faculty in the Department of Mathematics and Statistics at McMaster University redesigned their first-year calculus course to address student underpreparedness and use strategies such as diagnostic testing and bridging programs to place and support students enrolling in courses such as Calculus I (Kajander & Lovric, 2004). Additionally, secondary school counselors, if made aware of the advantages a high school calculus course could provide, would be in the position to advise students accordingly.

Limitations

The results obtained in our study are for one class, in one semester, at one institution only. Furthermore, locally developed calculus courses are not all equal, nor are the Mathematics 12 grades. The calculus backgrounds of the students may differ substantially, although recent western Canadian secondary school curricular reform efforts have provided a framework for a provincial Calculus 12 course. And even though research supports the use of Mathematics 12 grades as valuable indicators of mathematical ability, other factors can impact grades. Nevertheless, in spite of the limitations, we found evidence of significant differences in mean final grades between the two groups, those with antecedent calculus course work and those without antecedent calculus course work, favoring those with antecedent calculus. Our results clearly substantiate the importance of antecedent course work in calculus for improving grades in a course such as Calculus I. Given the serious consequences of lowered grades for students (potential loss of scholarship, decreased confidence, possible change of program, retention), additional analyses are essential to verify findings and to seek solutions to address inequities.

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