**The University of Northern British Columbia**
**College of Science and Management**
**Mathematics Program**

# MATHEMATICS 321 – TOPOLOGY
## Course Outline

Fall 2008

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**Prerequisites.** Math 224 – or a solid background in set theory and mathematical proofs. (Recommended prerequisite: Math 302, Theory of Metric Space.)

**Textbook.** TOPOLOGY by James R. Munkres, *Second Edition*, Prentice-Hall, Inc., ©2000.

**Course Description.** Open and closed sets, Hausdorff and other topologies, bases and sub-bases, contimuous functions, connectivity, product and quotient spaces, the Tychonoff and Uryshon Lemmas, metrization, compact spaces.

**Grading Scheme.** Your final grade $\Gamma$ will be calculated according to the weight function

$$\Gamma = .25\mathcal{A} + .2\mathcal{T}_1 + .2\mathcal{T}_2 + .35\Phi,$$

where $\mathcal{A}$ is the average of your assignment grades, $\mathcal{T}_1, \mathcal{T}_2$ are your grades for tests 1 and 2, and $\Phi$ is your final examination grade (all grades being in percentages).

**Test Dates.**

- Test 1: Wednesday October 8, 2008
- Test 2: Wednesday November 5, 2008
- Final Exam: To be set by the UNBC Registrar's Office

**Suggested Text Problems.**

| CHAPTER 1 | CHAPTER 2 | CHAPTER 3 |
|---|---|---|
| §1: 2aefimo 4 5 7 8 10 | §13: 1 2 3 4 7 8 | §23: 1 2 3 4 7 9 |
| §2: 1 2abcg 4abcd | §16: 1 3 4 6 9 | §24: 1 3 7a |
| §3: 1 2 3 4 5a 9 13 | §17: 1 2 3 6 8 10 13 | §26: 1a 2 3 4 5 6 8 |
| §5: 1 2 3a 4abc 5ac | §18: 1 2 3 6 7a 8 10 | |
| §7: 1 2 3 4a 5a-f | §19: ——— | |
| | §20: 1a 2 3 4 11 | |
| | §21: 1 2 5 6 7 8 9 | |

Additional topics may be included, time permitting.

It is important that students taking this course do all the problems listed so as to gain necessary background in the course (and to do them well). They, or similar problems, could appear in tests and/or final.

**Course Philosophy.**

Topology is the study of properties of geometric objects like surfaces and solid objects that are unchanged by elastic deformations such as twisting, stretching, compressing—but without cutting, tearing, or collapsing. This does not mean that we cannot cut, tear, or collapse an object. In fact, in Topology we often cut, tear, and collapse objects in order to construct new ones with different topologies. For example, if you glue the 11 inch ends of an 8.5 by 11 inch paper you will get a cylinder, which we say has a different topology from the original paper. If you glue the circular ends of this cylinder you will get yet another object, a doughnut or torus as we say, which is quite different in topology from the cylinder and the paper. We see that from very simple spaces (like a flat sheet of paper) we can construct more complex objects with quite different properties.

Topology also lays down the foundation for the notions of limits, continuity, approximation, convergence, in a very general setting.

In Topology one also studies properties that are invariant (i.e., unchanged) under *deformations* of a set. For example, a square can be deformed continuously into a circle—so they have the same topology. Also, the open interval $(0, 1)$ can be stretched to the interval $(-10, 10)$, and it can be stretched indefinitely to the whole real line $\mathbb{R}$. In other words, they all also have the same topology. Such spaces are said to be *homeomorphic* to each other.

The circle cannot be deformed into an interval without cutting it or collapsing parts of it, so they are not homeomorphic.

Topology also provides us with a very general setting for the notion of continuous functions which we learned about in Calculus. We will generalize some basic facts in Calculus such as the "maximum principle", which says that a continuous function over a *closed* interval $[a, b]$ attains its maximum value at a point in $[a, b]$. (By contrast, for the open interval $(0, 1)$ you can have a continuous function like $f(x) = 1/x$ that has no maximum.) The closed interval $[a, b]$ is going to be a special case of what we call a *compact* set in topology. So this "maximum principle" remains valid for continuous functions defined over compact topological spaces.

This course is, by its very nature, theoretically oriented and proof-based. It is based on careful logical reasoning (mostly deductive). But we will illustrate many ideas in geometric and intuitive terms (as much as and whenever possible) in order to gain some visual knowledge of them.

One of the biggest problems in topology, called the Poincare Conjecture, has recently been solved by the Russian mathematician Grigori Perelman in 2002/2003. He was awarded the Fields Medal for his work (this is the "Nobel Prize" in mathematics). This century old problem states that the 3-dimensional sphere is the only 3-dimensional compact space on which all closed loops on it can be shrunk to a point within the space (i.e., simply connected). For the 2-dimensional sphere the corresponding result is true and not difficult, but for 3-dimensions it proved to be quite a difficult problem.