STUDENT NAME: MASTER SHEET
STUDENT NUMBER:

MATHEMATICS 152 — CALCULUS!

Test 3

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[Marks]

50 minutes

Work in any order and answer all questions clearly – no calculators permitted.

[35] = **Total Marks.** Numbers in brackets [ ] are marks for each question.

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[6] 1. Find the derivative of each function.

(a) $y = 5^x - x^5$

$$y' = 5^x(\ln 5) - 5x^4$$

(b) $y = 8^{3x-2x^3}$

$$y' = 8^{3x-2x^3} \cdot (3-6x^2) \cdot (\ln 8)$$

(c) $y = \ln(x)\log_3(x)$

$$y' = \frac{1}{x} \cdot \log_3(x) + \ln(x) \cdot \frac{1}{x \cdot \ln 3}$$

[4] 2. Use logarithmic differentiation to find the derivative of the function $y = (x+1)^{\sqrt{x}}$. (Note: this is $1+x$ to the **power** $\sqrt{x}$.)

$$\ln y = \sqrt{x} \ln(x+1)$$

$$\therefore \quad \frac{1}{y} y' = \frac{1}{2} x^{-1/2} \ln(x+1) + \frac{\sqrt{x}}{x+1}$$

$$\therefore \quad y' = (x+1)^{\sqrt{x}} \cdot \left[\frac{1}{2\sqrt{x}} \ln(x+1) + \frac{\sqrt{x}}{x+1} \right]$$

3. Find the following integrals:

[3] (a) $\int (\frac{x^4 - 2}{x^2} + \frac{3}{x})\, dx$ $= \int \left(x^2 - 2x^{-2} + \frac{3}{x}\right) dx$

$$= \frac{x^3}{3} + \frac{2}{x} + 3 \ln|x| + C$$

[4] (b) $\int e^{3x+4}\, dx$

Let $u = 3x + 4$ (Substitution)

$du = 3dx$

$$\int e^u \cdot \frac{1}{3} du = \frac{1}{3} e^u + C = \frac{1}{3} e^{3x+4} + C$$

[5] (c) $\int_{-1}^{1} (x^4 + \sqrt[3]{x})\, dx$

$$= \left[\frac{x^5}{5} + \frac{x^{4/3}}{4/3}\right]_{-1}^{1} = \left[\frac{x^5}{5} + \frac{3}{4}x^{4/3}\right]_{-1}^{1}$$

$$= \left(\frac{1}{5} + \cancel{\frac{3}{4}}\right) - \left(-\frac{1}{5} + \cancel{\frac{3}{4}}\right)$$

$$= \frac{2}{5}$$

[4] (d) $\int \frac{x}{\sqrt{1-x^2}}\, dx$

Let $u = 1 - x^2$

$du = -2x\, dx$

$\therefore -\frac{1}{2} du = x\, dx$

$$= \int \frac{1}{\sqrt{u}} \cdot \left(\frac{-1}{2} du\right) = -\frac{1}{2}\int u^{-1/2}\, du$$

Power Rule

$$= -\frac{1}{2} \frac{u^{1/2}}{(1/2)} + C = -\sqrt{u} + C$$

$$= -\sqrt{1-x^2} + C$$

[4] (e) $\int x \ln x \, dx$

Integrate by Parts:

$u = \ln x \qquad dv = x\,dx$

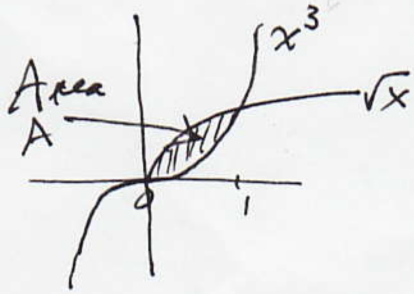
$du = \frac{1}{x}\,dx \qquad v = \frac{x^2}{2}$

$$\int x \ln x\,dx = \int u\,dv = uv - \int v\,du = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx$$

$$= \frac{x^2}{2}\ln(x) - \frac{1}{2}\int x\,dx$$

$$= \boxed{\frac{x^2}{2}\ln(x) - \frac{x^2}{4} + C}$$

[5] 4. Find the area of the region bounded by the graphs of $y = x^3$ and $y = \sqrt{x}$. Sketch the region.



$$A = \int_0^1 \left(\sqrt{x} - x^3\right)dx$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{x^4}{4}\right]_0^1$$

$$= \frac{2}{3} - \frac{1}{4}$$

$$= \frac{5}{12}$$ is the area.

$\sqrt{x} = x^3$

$\Rightarrow x = 0, 1$ crossing points.

End